
Mathematics
Higher
Paper 2
Practice Paper E

Time allowed
1 hour 10 minutes

NATIONAL
QUALIFICATIONS

Read carefully

- 1 **Calculators may be used in this paper.**
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product : $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

$$\text{or } \mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3, \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives :

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals :

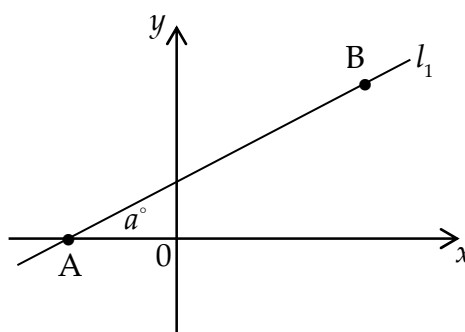
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

1. (a) A line, l_1 , passes through the points A(-3, 0) and B(5, 4).

The line makes an angle of a° with the positive direction on the x -axis.

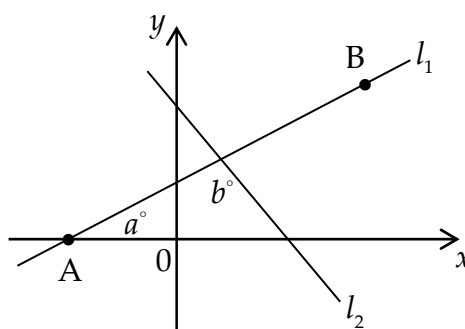


Find the value of a .

3

- (b) A second line, l_2 , with equation $4x + 3y = 12$, crosses the line in (a).

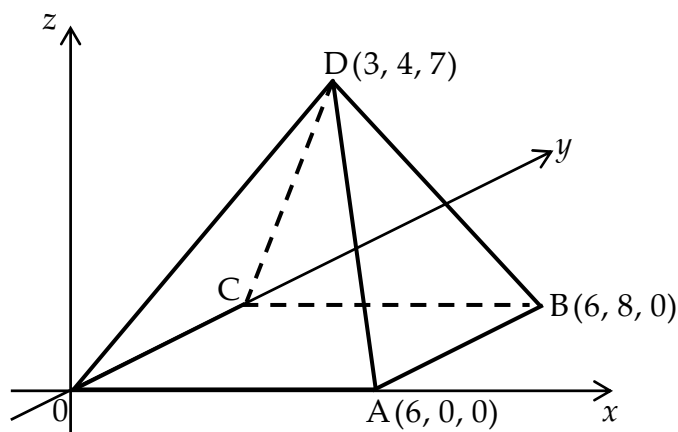
The angle between the two lines is b° , as shown.



Find the value of b .

4

2. The rectangular based pyramid D,OABC has vertices A(6, 0, 0), B(6, 8, 0) and D(3, 4, 7).



- (a) (i) Write down the coordinates of C.
 (ii) Express \overrightarrow{AC} and \overrightarrow{AD} in component form.
- (b) Calculate the size of angle CAD.

3

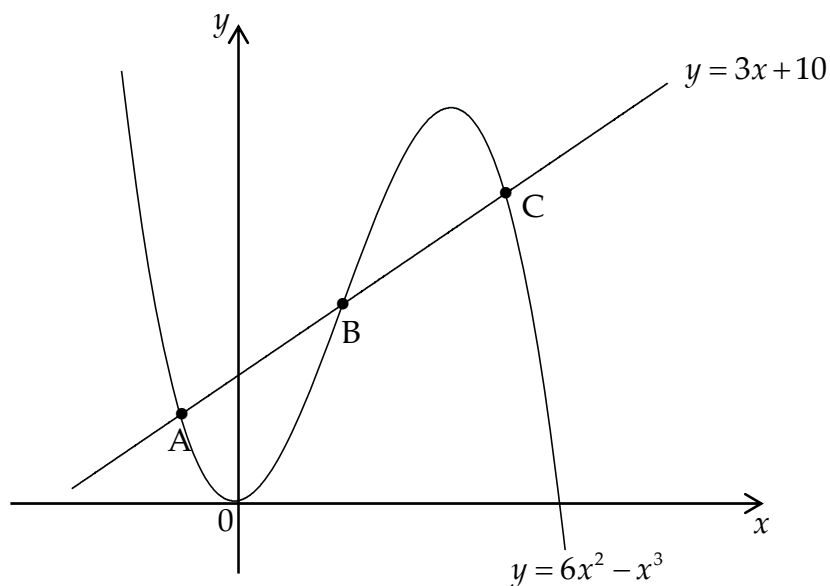
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3. (a) (i) Show that $(x-2)$ is a factor of $x^3 - 6x^2 + 3x + 10$.

(ii) Hence factorise $x^3 - 6x^2 + 3x + 10$ fully.

4

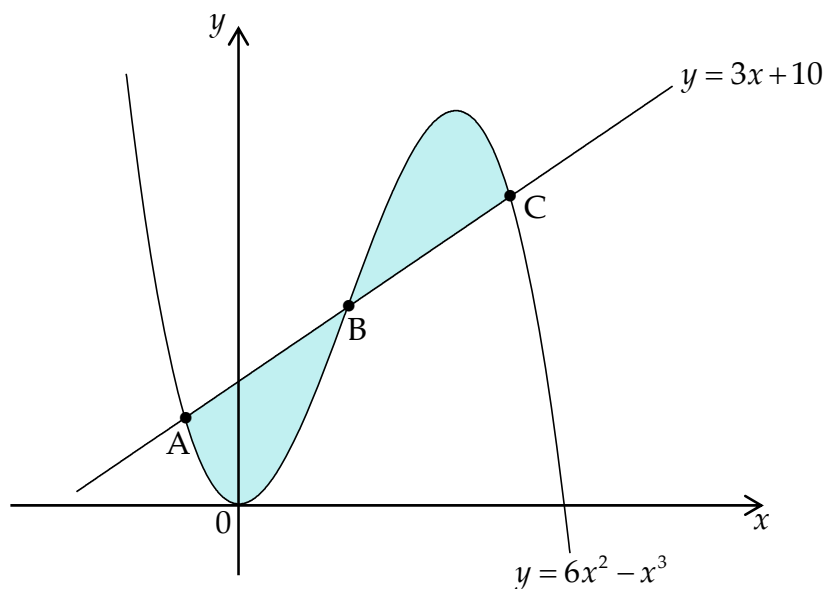
The line with equation $y = 3x + 10$ intersects the curve with equation $y = 6x^2 - x^3$ at the points A, B and C.



(b) Find the x -coordinates of the points A and C.

3

The area between the curve and the line from A to C is shaded in the diagram below.



(c) Calculate the total shaded area shown in the diagram.

7

4. Solve $2 \cos 2x - \sin x + 1 = 0$ for $0 \leq x < 2\pi$. 6

5. A new '24 hour anti-biotic' is being tested on a patient in hospital.

It is known, that over a 24 hour period, the amount of anti-biotic remaining in the bloodstream is reduced by 80%.

On the first day of the trial, an initial 250 mg dose is given to a patient at 7 a.m.

(a) After 24 hours and just prior to the second dose being given, how much anti-biotic remains in the patient's bloodstream? 1

The patient is then given a further 250 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(b) A recurrence relation of the form $u_{n+1} = au_n + b$ can be used to model this course of treatment.

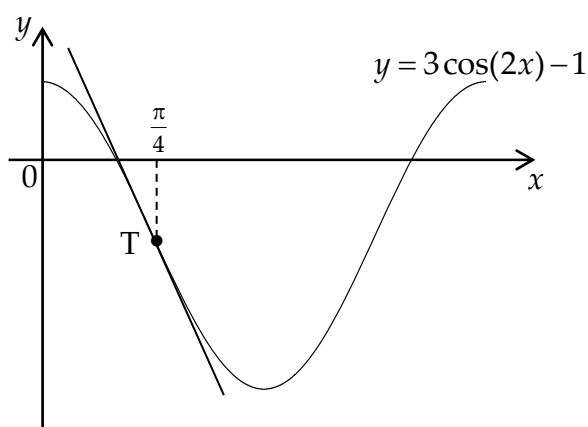
Write down the values of a and b . 2

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(c) Is it safe to administer this anti-biotic over an extended period of time? 4

6. The diagram shows part of the graph of $y = 3 \cos(2x) - 1$.

Find the equation of the tangent at the point T, where $x = \frac{\pi}{4}$.



7

7. Solve $\log_x(x+2) + \log_x(2x-3) = 2$, $x > \frac{3}{2}$. 5

8. A circle has the following properties:

- The x -axis and the line $y = 20$ are tangents to the circle.
- The circle passes through the points $(0, 2)$ and $(0, 18)$.
- The centre lies in the first quadrant.

Find the equation of this circle.

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End of Question Paper