



# National Higher Mathematics

---

Sample Questions D

Paper 2  
Calculator

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product :**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ , where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives :**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals :**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

1. A sequence is defined by recurrence relation  $u_{n+1} = ku_n - 6$ ,  $u_0 = 0$ .

(a) Given that  $u_2 = -8$ , find the value of  $k$ . 2

(b) (i) Why does this sequence tend to a limit as  $n \rightarrow \infty$ ?

(ii) Find the value of this limit. 3

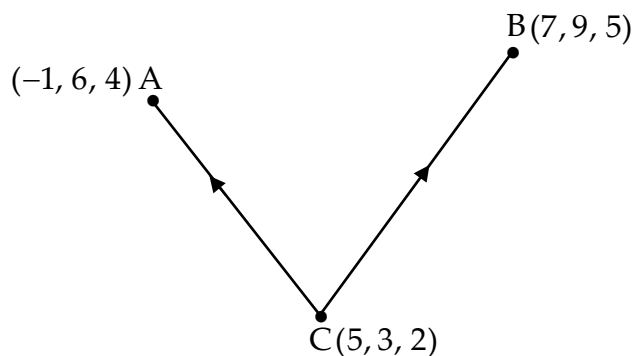
2.  $f(x) = 2x^3 + px^2 + qx + 4$ .

Given that  $(x - 2)$  is a factor of  $f(x)$ , and the remainder when  $f(x)$  is divided by  $(x + 1)$  is 9, find the values of  $p$  and  $q$ . 5

3. Security guards are watching a parked car, via two CCTV cameras, in a supermarket car park.



With reference to a suitable set of axes, the car is at  $C(5, 3, 2)$  and the cameras are at positions  $A(-1, 6, 4)$  and  $B(7, 9, 5)$  as shown.

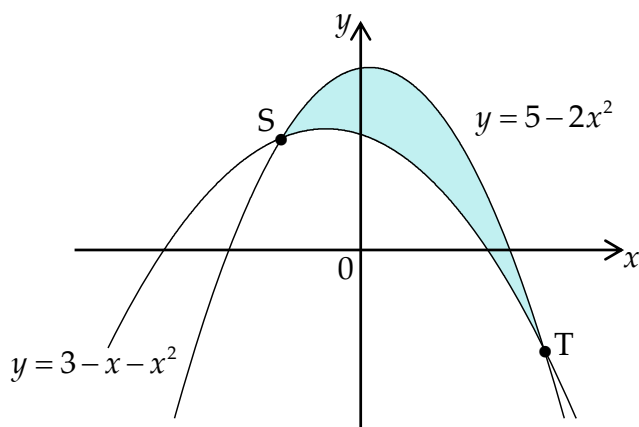


Calculate the size of angle  $ACB$ .

7

4. Part of the graphs of  $y = 3 - x - x^2$  and  $y = 5 - 2x^2$  are shown opposite.

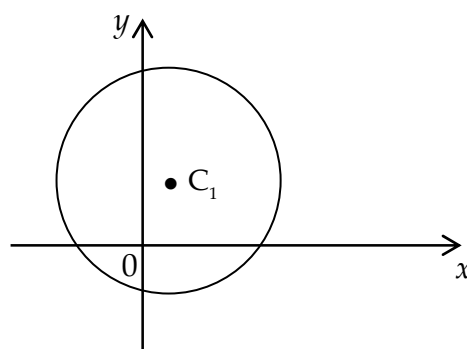
The curves intersect at the points S and T.



- (a) Find the coordinates of S and T. 4
- (b) Find the shaded area enclosed between the two curves. 5

5. A circle with centre  $C_1$  has equation  $x^2 + y^2 - 2x - 6y - 15 = 0$ .

- (a) Write down the coordinates of the centre and calculate the length of the radius of this circle.

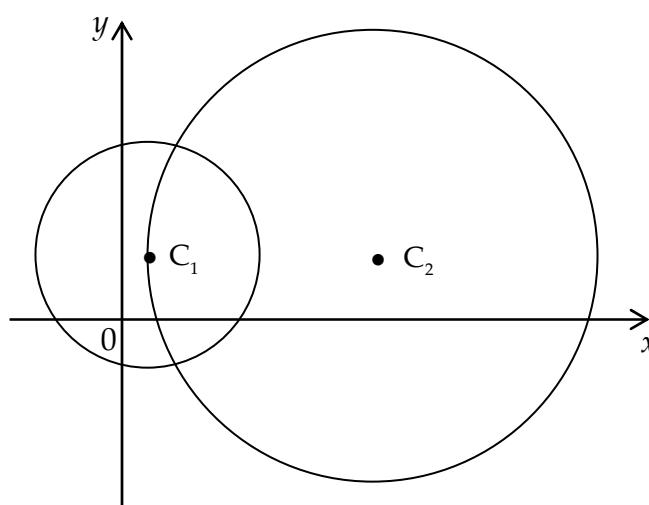


2

A second circle with centre  $C_2$  has a diameter twice that of the circle with centre  $C_1$ .

$C_1$  lies on the circumference of this second circle.

The line joining  $C_1$  and  $C_2$  is parallel to the  $x$ -axis.



- (b) Find the equation of the circle with centre  $C_2$ . 4

6. A manufacturer of executive desks estimates that the weekly cost, in £, of making  $x$  desks is given by  $C(x) = x^3 - 6x^2 + 560x + 800$ .

Each executive desk sells for £2000.



- (a) Show that the weekly profit made from making  $x$  desks is given by

$$P(x) = -x^3 + 6x^2 + 1440x - 800 \quad 2$$

- (b) (i) How many desks would the manufacturer have to make each week in order to maximise his profit?
- (ii) What would his annual profit be? 8

7. The number of bacteria,  $b$ , in a culture after  $t$  hours is given by  $b = b_0 e^{kt}$  where  $b_0$  is the original number of bacteria present.



- (a) The number of bacteria in a culture increases from 800 to 2400 in 2 hours. Find the value of  $k$  correct to 3 significant figures. 3

- (b) How many bacteria, to the nearest hundred, are present after a further 4 hours? 3

8. (a) Express  $2 \cos x^\circ - 5 \sin x^\circ$  in the form  $k \cos(x+a)^\circ$ , where  $k > 0$  and  $0 < a < 90$ . 4
- (b) (i) Hence write  $2 \cos 2x^\circ - 5 \sin 2x^\circ$  in the form  $R \cos(2x+b)^\circ$ , where  $R > 0$  and  $0 < b < 90$ . 2
- (ii) Solve  $2 \cos 2x^\circ - 5 \sin 2x^\circ = 5$  in the interval  $0 \leq x < 360$ . 4

**End of Question Paper**