

Higher Grade Paper 2 2010/2011

Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark															
1(a)	<p>ans: $3y = x + 15$ (3 marks)</p> <ul style="list-style-type: none"> •¹ finds midpoint of BC •² finds gradient of AM •³ subs into equation of straight line 	<ul style="list-style-type: none"> •¹ midpoint BC = (9, 8) •² $m_{AM} = \frac{1}{3}$ •³ $y - 8 = \frac{1}{3}(x - 9)$ or $y = \frac{1}{3}x + 5$ 															
(b)	<p>ans: A(-9, 2) (3 marks)</p> <ul style="list-style-type: none"> •¹ knows to use system of equations •² solves for x and y •³ states coordinates of E 	<ul style="list-style-type: none"> •¹ evidence •² $x = -9 ; y = 2$ •³ A(-9, 2) 															
(c)	<p>ans: $y = 5x - 17$ (3 marks)</p> <ul style="list-style-type: none"> •¹ finds gradient of AC •² finds gradient of altitude •³ subs into equation of straight line 	<ul style="list-style-type: none"> •¹ $m_{AB} = -\frac{1}{5}$ •² $m_{alt} = 5$ •³ $y - 18 = 5(x - 7)$ 															
2(a)	<p>ans: $k = 6$ (3 marks)</p> <ul style="list-style-type: none"> •¹ knows to use synthetic division •² uses synthetic division correctly •³ equates remainder to 0 and solves for k 	<ul style="list-style-type: none"> •¹ evidence <table style="margin-left: 40px;"> <tr> <td>1</td> <td>3</td> <td>k</td> <td>4</td> <td>-13</td> </tr> <tr> <td></td> <td></td> <td>3</td> <td>k+3</td> <td>k+7</td> </tr> <tr> <td></td> <td>3</td> <td>k+3</td> <td>k+7</td> <td>k-6</td> </tr> </table> <ul style="list-style-type: none"> •² •³ $k - 6 = 0; k = 6$ 	1	3	k	4	-13			3	k+3	k+7		3	k+3	k+7	k-6
1	3	k	4	-13													
		3	k+3	k+7													
	3	k+3	k+7	k-6													
(b)	<p>ans: $x = -\frac{2}{3}$ (4 marks)</p> <ul style="list-style-type: none"> •¹ finds derivative •² makes derivative equal to 0 for SP •³ factorises •⁴ solves for x 	<ul style="list-style-type: none"> •¹ $\frac{dy}{dx} = 9x^2 + 12x + 4$ •² $9x^2 + 12x + 4 = 0$ at SP •³ $(3x + 2)(3x + 2) = 0$ •⁴ $x = -\frac{2}{3}$ 															

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3(a)	<p>ans: $a = 9p^2; b = -6p; c = \frac{1}{2}p$ (4 marks)</p> <ul style="list-style-type: none"> •¹ knows to substitute •² substitutes correctly •³ simplifies to correct form •⁴ states values of a, b and c 	<ul style="list-style-type: none"> •¹ evidence of sub. one function into other •² $3p[x(3px - 2)] + \frac{1}{2}p$ •³ $3p(3px^2 - 2x) + \frac{1}{2}p; 9p^2x^2 - 6px + \frac{1}{2}p$ •⁴ $a = 9p^2; b = -6p; c = \frac{1}{2}p$
(b)	<p>ans: $p = 2$ (3 marks)</p> <ul style="list-style-type: none"> •¹ knows discriminant = 0 •² substitutes values and simplifies •³ solves and discards 	<ul style="list-style-type: none"> •¹ $b^2 - 4ac = 0$ [stated or implied] •² $(-6p)^2 - 4 \times 9p^2 \times \frac{1}{2}p = 0; 36p^2 - 18p^3 = 0$ •³ $18p^2(2 - p) = 0; p = 2$
4(a)	<p>ans: P(3, 12); Q(4, 0) (5 marks)</p> <ul style="list-style-type: none"> •¹ for P: knows to equate functions •² finds x – coord. of P •³ for Q: equates function to 0 •⁴ solves for x •⁵ states coords. of P and Q 	<ul style="list-style-type: none"> •¹ $x^3 - 11x^2 + 28x = 4x$ •² $x = 3$ •³ $x^3 - 11x^2 + 28x = 0$ •⁴ $x = 4$ •⁵ P(3, 12); Q(4, 0)
(b)	<p>ans: $24\frac{1}{12}$ units² (6 marks)</p> <ul style="list-style-type: none"> •¹ splits area into 2 sections •² finds area of triangle •³ sets up integral for other area •⁴ finds integral •⁵ subs values •⁶ evaluates and adds to area of triangle 	<ul style="list-style-type: none"> •¹ evidence of area of triangle plus integral •² $\frac{1}{2}(3 \times 12) = 18$ units² •³ $\int_3^4 x^3 - 11x^2 + 28x \, dx$ •⁴ $\left[\frac{x^4}{4} - \frac{11x^3}{3} + 14x^2 \right]_3^4$ •⁵ $\left[\frac{4^4}{4} - \frac{11(4)^3}{3} + 14(4)^2 \right] - \left[\frac{3^4}{4} - 11(3)^3 + 14(3)^2 \right]$ •⁶ $6\frac{1}{12} + 18 = 24\frac{1}{12}$ units²
Alternative solution is given on next page		
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	Give 1 mark for each •	Illustration(s) for awarding each mark
or(b)	<p>ans: $24\frac{1}{12}$ units² (6 marks)</p> <ul style="list-style-type: none"> •¹ knows to find area under curve •² finds integral •³ subs values •⁴ evaluates •⁵ finds unshaded area •⁶ subtracts to answer 	<ul style="list-style-type: none"> •¹ $\int_0^4 x^3 - 11x^2 + 28x \, dx$ •² $\left[\frac{x^4}{4} - \frac{11x^3}{3} + 14x^2 \right]_0^4$ •³ $\left[\frac{4^4}{4} - \frac{11(4)^3}{3} + 14(4)^2 \right] - 0$ •⁴ $160/3$ •⁵ $\left[\frac{x^4}{4} - \frac{11x^3}{3} + 12x^2 \right]_0^3 = \frac{117}{4}$ •⁶ $\frac{160}{3} - \frac{117}{4} = 24\frac{1}{12}$ units²
5(a)	<p>ans: $p = 6$ (2 marks)</p> <ul style="list-style-type: none"> •¹ subs into equation of circle •² solves for p 	<ul style="list-style-type: none"> •¹ $p^2 + 144 - 192 + 12 = 0$ •² $p^2 = 36; p = 6$
(b)	<p>ans: $(0, -8)$ (1 mark)</p> <ul style="list-style-type: none"> •¹ states centre of circle 	<ul style="list-style-type: none"> •¹ $(0, -8)$
(c)	<p>ans: $2y = 3x - 42$ (4 marks)</p> <ul style="list-style-type: none"> •¹ finds gradient of ST •² finds gradient of tangent •³ subs into equation of straight line •⁴ finds coords of point R 	<ul style="list-style-type: none"> •¹ $m_{ST} = -2/3$ •² $m_{tan} = \frac{3}{2}$ •³ $y + 12 = \frac{3}{2}(x - 6)$ [or equivalent] •⁴ $3x - 42 = 0; x = 14$ $(14, 0)$
(d)	<p>ans: $(x - 7)^2 + (y + 4)^2 = 65$ (3 marks)</p> <ul style="list-style-type: none"> •¹ finds midpoint of SR (centre of circle) •² finds radius •³ subs into equation of circle 	<ul style="list-style-type: none"> •¹ centre of circle $(7, -4)$ •² $\sqrt{65}$ •³ $(x - 7)^2 + (y + 4)^2 = 42$
6	<p>ans: $a = 2$ (5 marks)</p> <ul style="list-style-type: none"> •¹ knows to make derivative equal to 0 •² prepares to differentiate •³ finds derivative •⁴ attempts to solve for a •⁵ solves for a 	<ul style="list-style-type: none"> •¹ $C' = 0$ [stated or implied] •² $C = -\frac{16200}{9}a^{-1} + 450a$ •³ $C' = -\frac{16200}{9a^2} + 450 = 0$ •⁴ $-\frac{16200}{9a^2} = -450; 4050a^2 = 16200$ •⁵ $a^2 = 4; a = 2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	ans: $p = -\frac{1}{4}; q = -\frac{17}{16}$ (2 marks) <ul style="list-style-type: none"> •¹ completes square •² states values of p and q 	<ul style="list-style-type: none"> •¹ $(\sin \theta - \frac{1}{4})^2 - \frac{17}{16}$ •² $p = -\frac{1}{4}; q = -\frac{17}{16}$
(b)	ans: 0.25 radians (3 marks) <ul style="list-style-type: none"> •¹ states minimum value •² attempts to find θ •³ finds value of θ 	<ul style="list-style-type: none"> •¹ minimum value = $-\frac{17}{16}$ •² $\sin \theta = \frac{1}{4}$ •³ $\theta = 0.25$ radians
8(a)	ans: $f(x) = x^3 + \frac{10}{x} - 10$ (5 marks) <ul style="list-style-type: none"> •¹ knows to integrate •² integrates first term correctly •³ 2nd term integrated correctly + C •⁴ equates from additional information •⁵ solves for C and states $f(x) = \dots$ 	<ul style="list-style-type: none"> •¹ $f(x) = \int f'(x) dx$ [stated or implied] •² $= \frac{3x^3}{3} \dots\dots\dots$ •³ $= \dots\dots\dots - \frac{10x^{-1}}{-1} + C$ •⁴ $3 = 2^3 + \frac{10}{2} + C$ •⁵ $\therefore C = -10$ and $f(x) = x^3 + \frac{10}{x} - 10$
(b)	ans: $f(1) = 1$ (1 mark) <ul style="list-style-type: none"> •¹ calculates to answer 	<ul style="list-style-type: none"> •¹ $f(1) = 1^3 + \frac{10}{1} - 10 = 1$
		Total: 60 marks