## Higher Grade Paper 2 2009/2010

**Marking Scheme** 

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $a = 1; b = 4, c = -29$ (4 marks) • <sup>1</sup> finds gradient of BD • <sup>2</sup> finds gradient of AC • <sup>3</sup> subs into $y - b = m(x - a)$ and rearranges • <sup>4</sup> states values of <i>a</i> , <i>b</i> and <i>c</i>	• $m_{BD} = 4$ [from equation] • $m_{AC} = -\frac{1}{4}$ • $y - 8 = -\frac{1}{4}(x+3); x + 4y - 29 = 0$ • $a = 1; b = 4, c = -29$
(b)	ans: $E(5, 6)$ (3 marks)• $^1$ knows to use system of equations• $^2$ solves for x and y• $^3$ states coordinates of E	• 1 evidence of equating one variable • 2 $x = 5; y = 6$ • 3 $E(5, 6)$
(c)	ans: C(13, 4) (2 marks)	<ul> <li>evidence of 'stepping out' or other method</li> <li>C(13, 4)</li> </ul>
2(a) (b)	ans:proof(3 marks) $\cdot^1$ knows to substitute $\cdot^2$ substitutes correctly $\cdot^3$ clearly simplifies to answerans: $p = 2$ (4 marks) $\cdot^1$ substitute for $x$ $\cdot^1$ substitute for $x$ $\cdot^2$ knows to multiply by conjugate surd $\cdot^3$ multiplies and simplifies $\cdot^4$ states value of $p$	• 1 evidence of sub. one function in other • 2 $f\left(\frac{1}{x-1}\right) = \frac{4}{x-1} + 1$ • 3 $\frac{4+x-1}{x-1} = \frac{x+3}{x-1}$ • 1 $\frac{\sqrt{5}+3}{\sqrt{5}-1}$ • 2 $\frac{\sqrt{5}+3}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$ • 3 $\frac{5+4\sqrt{5}+3}{4} = \frac{8+4\sqrt{5}}{4} = 2+\sqrt{5}$ • 4 $p=2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	ans: $P(1, 0); Q(-2, 27)$ (5 marks)•1knows derivative = 0 at S.P.•2takes derivative and factorises•3solves for x and chooses appropriate value•4substitutes to find $y$ – coordinate•5states coordinates of P and Q	• <sup>1</sup> $f'(x) = 0$ at SP [stated or implied] • <sup>2</sup> $6x^2 + 6x - 12 = 0$ ; $6(x+2)(x-1) = 0$ • <sup>3</sup> $x = -2$ or 1 • <sup>4</sup> $f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 7 = 27$ • <sup>5</sup> P(1, 0); Q(-2, 27)
(b)	<ul> <li>ans: 40.5 units<sup>2</sup> (4 marks)</li> <li>•<sup>1</sup> sets up integral</li> <li>•<sup>2</sup> integrates expression</li> <li>•<sup>3</sup> substitutes values</li> <li>•<sup>4</sup> evaluates</li> </ul>	• <sup>1</sup> $\int_{-2}^{1} 2x^3 + 3x^2 - 12x + 7  dx$ • <sup>2</sup> $\left[\frac{x^4}{2} + x^3 - 6x^2 + 7x\right]_{-2}^{1}$ $\left(\frac{(1)^4}{2} + (1)^3 - 6(1)^2 + 7(1)\right) - \left(\frac{(-2)^4}{2} + (-2)^3 - 6(-2)^2 + 7(-2)\right)$ • <sup>4</sup> 40.5 units <sup>2</sup>
4	<ul> <li>ans: 30°,150°,270°. (5 marks)</li> <li><sup>1</sup> Re-arranges equation</li> <li><sup>2</sup> factorises</li> <li><sup>3</sup> states solution for sinx</li> <li><sup>4</sup> finds two solutions</li> <li><sup>5</sup> finds further solution</li> </ul>	• $2\sin^2 x + \sin x - 1 = 0$ • $(2\sin x - 1)(\sin x + 1)$ • $\sin x = \frac{1}{2}$ , AND $\sin x = -1$ • $x = 30^{\circ}$ and $150^{\circ}$ • $x = 270^{\circ}$

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5(a)	ans: proof (2 marks)	
	<ul> <li><sup>1</sup> substitutes U<sub>0</sub> and finds U<sub>1</sub></li> <li><sup>2</sup> substitutes U<sub>1</sub> and finds U<sub>2</sub></li> </ul>	• $U_1 = \frac{a}{4} \times 16 + 12 = 4a + 12$ • $U_1 = \frac{a}{4}(4a + 12) + 12 = a^2 + 3a + 12$
(b)	ans: $a = 3$ (3 marks) • <sup>1</sup> equates $U_2$ to 30 • <sup>2</sup> collects terms to LHS and factorises • <sup>3</sup> solves for x and discards	• <sup>1</sup> $a^{2} + 3a + 12 = 30$ • <sup>2</sup> $a^{2} + 3a - 18 = 0; (a + 6)(a - 3) = 0$ • <sup>3</sup> $a = -6, 3; a = 3$
(c)	<ul> <li>ans: 48 (3 marks)</li> <li>•<sup>1</sup> knows condition for limit</li> <li>•<sup>2</sup> knows how to find limit</li> <li>•<sup>3</sup> answer</li> </ul>	• 1 limit exists since $-1 < \frac{3}{4} < 1$ • 2 $L = \frac{12}{1 - 0.75} = \frac{12}{0.25}$ • 3 48
6(a)	ans: $y = 2x$ (3 marks) • <sup>1</sup> finds midpoint of QR • <sup>2</sup> finds gradient of PA • <sup>3</sup> substitutes in $y-b = m(x-a)$	• midpoint of QR = (2, 4) • $m_{PA} = \frac{4+6}{2+3} = 2$ • $y-4 = 2(x-2); y = 2x$
(b) (c)	ans: C(7, 14) (4 marks) • <sup>1</sup> knows to substitute line into circle • <sup>2</sup> multiplies and simplifies • <sup>3</sup> factorises and solves • <sup>4</sup> chooses appropriate value for x and subs ans: $(x-7)^2 + (y-14)^2 = 5$ (3 marks)	• $x^{2} + (2x)^{2} - 10x - 20(2x) + 105 = 0$ • $5x^{2} - 50x + 105 = 0$ • $5(x-3)(x-7) = 0$ • $x = 3,7; x = 7, y = 14$
	• 1 finds radius of larger circle • 2 finds radius of smaller circle • 3 subs into $(x - a)^2 + (y - b)^2 = r^2$	• radius (large) = $\sqrt{25+100-105} = \sqrt{20}$ • radius (small) = $\sqrt{5}$ • $(x-7)^2 + (y-14)^2 = 5$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	ans: $k = 2$ (3 marks) • <sup>1</sup> knows to use synthetic division • <sup>2</sup> makes remainder = 0 • <sup>3</sup> solves for k	• <sup>1</sup> evidence • <sup>2</sup> $8-4k=0$ • <sup>3</sup> $k=2$
(b)	ans: $p = -3$ (3 marks) • <sup>1</sup> equates function to 35 • <sup>2</sup> collect terms to LHS and equates to 0 • <sup>3</sup> uses synthetic division to find root	• $p^{3} - 2p^{2} - 16p + 32 = 35$ • $p^{3} - 2p^{2} - 16p - 3 = 0$ • $p^{3} - 2p^{2} - 16p - 3 = 0$
(c)	<ul> <li>ans: 98° (2 marks)</li> <li><sup>1</sup> finds gradient of AB</li> <li><sup>2</sup> takes tan <sup>-1</sup> and states angle</li> </ul>	• $m_{AB} = \frac{35 - 0}{-3 - 2} = -7$ • $\tan^{-1}(7) = 82^{\circ}; \text{ angle} = 98^{\circ}$
8	<ul> <li>ans: a = 3 (4 marks)</li> <li>•<sup>1</sup> evaluates integral</li> <li>•<sup>2</sup> finds derivative</li> <li>•<sup>3</sup> makes integral = derivative</li> <li>•<sup>4</sup> factorises and solves</li> </ul>	• $[x^2]_0^a = a^2$ • $\frac{d}{da} = 6a - 9$ • $\frac{a^2}{a^2} = 6a - 9; a^2 - 6a + 9 = 0$ • $(a - 3)(a - 3) = 0; a = 3$ Total: 60 marks