Higher Grade Paper 2 2008/2009

Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $k = 6$ (2 marks) • ¹ knows to substitute point • ² establishes value of k	• ¹ $(0+4)^2 + k^2 = 52$ • ² $k = 6$
(b)	ans: $y = -\frac{2}{3}x + 6$ (4 marks)•1finds coordinates of C1•2finds gradient of radius•3finds gradient of tangent•4substitutes into formula	• $C(-4, 0)$ • $m_{C_1P} = \frac{6}{4} = \frac{3}{2}$ • $m_{tan} - \frac{2}{3}$ • $y = -\frac{2}{3}x + 6$
(c) (d)	ans: $C_2(9, 0)$ (1 mark)•1subs point, solves for x and states pointans: 2·2 units(3 marks)•1finds radius C_1 circle•2finds distance between centres•3establishes d	• $0 = -\frac{2}{3}x + 6; x = 9; (9, 0)$ • $1 \operatorname{radius} C_1 = 7 \cdot 2$ • $2 \operatorname{C}_1 C_2 = 13$ • $d = (7 \cdot 2 + 8) - 13 = 2 \cdot 2$
2	 ans: 60, 120, 240, 300 (5 marks) •¹ obtains composite •² equates to 0 •³ solves •⁴ finds two solutions •⁵ finds two solutions 	• $h(x)=4\cos^{2} x + 1$ • $4\cos^{2} x + 1 = 0$ • $\cos x = \frac{1}{2}$ and $\cos x = -\frac{1}{2}$ • $x = 60, 300$ • $x = 120, 240$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	ans: $y = x^2 + \frac{6}{x} - 4$ (4 marks)	
	\bullet^1 knows to integrate	• ¹ $y = \int 2x - \frac{6}{r^2} dx$
	\bullet^2 integrates	$\bullet^2 y = x^2 + \frac{6}{x} + C$
	\bullet^3 subs point	• $3 = 2^2 + \frac{6}{2} + C$
	• ⁴ solves for <i>C</i> and states function	$\bullet^4 y = x^2 + \frac{6}{x} - 4$
(b)	ans: $p = 7$ (1 mark)	
	• ¹ subs point and solves for p	• $p = 3^2 + \frac{6}{3} - 4 = 7$
4(a)	ans: P(3, 0) (2 marks)	
	• knows to make function equal to 0 • solves for x and states cords of P	• $x^{3} - x^{2} - 5x - 3 = 0$ • $x = 3; P(3, 0)$
(b)	ans: $2y + 3x = 9$ (1 mark)	
	\bullet^1 subs info into formula for straight line	• $y = -\frac{3}{2}(x-3)$
(c)	ans: $y - 11x = 17$ (4 marks)	
	\bullet^1 knows to take derivative	$\bullet^1 \qquad \frac{dy}{dx} = 3x^2 - 2x - 5$
	\bullet^2 subs to find gradient	• ² $3(-2)^2 - 2(-2) - 5 = 11$
	\bullet^3 subs to find point of contact	• ³ $y = (-2)^3 - (-2)^2 - 5(-2) - 3 = -5$
	• subs into straight line formula	• $y+5=11(x+2); y-11x=17$
(d)	ans: Q(-1, 6) (3 marks)	
	\bullet^1 knows to use sim. eqs.	\bullet^1 evidence
	• ² solves for x and y • ³ states coordinates of Q	• $x = -1$ and $y = 6$ • $Q(-1, 6)$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	ans: 143-3 <i>gu's</i> (2 marks)	
	 knows how to calculate answer answer 	• 1 0.92 ⁴ × 200 • 2 143.3gu's
(b)	ans: 135.8 gu's (3 marks)	
	 sets up recurrence relation repeated calculations to answer repeated calculations to answer 	• $U_{n+1} = 0.92^4 U_n + 32$ • 175.3[after 4 hours]; 157.6[after 8 hours] • 144.9[after 12 hours]; 135.8[after 16 hours]
(c)	ans: yes since lower limit is 80.8 (3 marks)	
	 ¹ knows to find limit ² finds limit ³ realises lower limit is less than 100 	• $L = \frac{32}{1-0.92^4}$ • $L = 112.8$ • $J = 112.8$ • $J = 112.8$ • $L = 12.8$ • $L = 12.8$
6(a)	ans: proof (3 marks)	
	 ¹ cross multiplies and multiplies out ² brings to LHS ³ rearranges as required 	• $k(x^{2} + 4) = x^{2} - 2x + 1$ • $kx^{2} - x^{2} + 2x + 4k - 1$ • $(k-1)x^{2} + 2x + (4k-1) = 0$
(b)	ans: $k = \frac{5}{4}$ (5 marks)	
	 ¹ states condition for equal roots states values of <i>a</i>, <i>b</i> and <i>c</i> ³ substitutes into b² - 4ac ⁴ multiplies out and simplifies ⁵ solves for <i>k</i> 	• $b^{2} - 4ac = 0$ for equal roots [stated/implied] • $a = (k-1); b = 2; c = (4k-1)$ • $2^{2} - 4(k-1)(4k-1) = 0$ • $20k - 16k^{2} = 0$ • $k = \frac{5}{4}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	ans: proof (3 marks)	
	 ¹ finds expression for length of shed ² finds expression for area of g'house ³ simplifies to correct form 	• length of shed $=\frac{3}{x}$ • $A = (x+3)(4+\frac{3}{x})-3$ • $A = 4x+3+12+\frac{9}{x}-3 \rightarrow \text{answer}$
(b)	ans: 15 (5 marks)	
	 ¹ knows to equate derivative to 0 ² prepares to differentiate ³ differentiates ⁴ solves for <i>x</i> ⁵ justifies answer 	• $\frac{dy}{dx} = 0$ • $A(x) = 12 + 4x + 9x^{-1}$ • $A'(x) = 4 - \frac{9}{x^2} = 0$ • $4 - \frac{9}{x^2} = 0; x^2 = \frac{9}{4}; x = \frac{3}{2}$ • y or other acceptable method
8(a)	ans: $\frac{\pi}{2}$ radians (6 marks) • 1 identifies coefficients • 2 identifies one factor • 3 correct second factor • 4 • 5 solves • 6 valid conclusion for $\sin^2\theta = -4$	• ¹ 1 -1 4 -4 • ² (sin θ -1) • ³ (sin ² θ + 4) • ⁴ sin θ = 1 and sin ² θ = -4 • ⁵ $\theta = \frac{\pi}{2}$ radians • ⁶ sin 2θ = -4 has no solution
		Total: 60 marks