Higher Grade Paper 2 2008/2009
Marking Scheme

|  | Give 1 mark for each - | Illustration(s) for awarding each mark |
| :---: | :---: | :---: |
| 1(a) <br> (b) <br> (c) <br> (d) | ans: $k=6$ <br> - ${ }^{1}$ knows to substitute point <br> - ${ }^{2}$ establishes value of $k$ <br> ans: $y=-\frac{2}{3} x+6$ <br> (4 marks) <br> - ${ }^{1}$ finds coordinates of $\mathrm{C}_{1}$ <br> - ${ }^{2}$ finds gradient of radius <br> -3 finds gradient of tangent <br> - ${ }^{4}$ substitutes into formula <br> ans: $\quad C_{2}(\mathbf{9}, 0)$ <br> (1 mark) <br> - ${ }^{1} \quad$ subs point, solves for $x$ and states point <br> ans: $\mathbf{2 . 2}$ units <br> (3 marks) <br> - ${ }^{1}$ finds radius $\mathrm{C}_{1}$ circle <br> $\bullet$ finds distance between centres <br> - ${ }^{3}$ establishes $d$ | - ${ }^{1}(0+4)^{2}+k^{2}=52$ <br> - ${ }^{2} \quad k=6$ <br> - ${ }^{1} \mathrm{C}(-4,0)$ <br> - $m_{C_{1} P}=\frac{6}{4}=\frac{3}{2}$ <br> - $m_{\mathrm{tan}}-\frac{2}{3}$ <br> - ${ }^{4} y=-\frac{2}{3} x+6$ <br> - ${ }^{1} \quad 0=-\frac{2}{3} x+6 ; x=9 ;(9,0)$ <br> - ${ }^{1}$ radius $\mathrm{C}_{1}=7 \cdot 2$ <br> - ${ }^{2} \quad \mathrm{C}_{1} \mathrm{C}_{2}=13$ <br> - ${ }^{3} d=(7 \cdot 2+8)-13=2 \cdot 2$ |
| 2 | ans: 60, 120, 240, 300 <br> (5 marks) <br> - ${ }^{1}$ obtains composite <br> - ${ }^{2}$ equates to 0 <br> - ${ }^{3}$ solves <br> -4 finds two solutions <br> -5 finds two solutions | - ${ }^{1} h(x)=4 \cos ^{2} x+1$ <br> - ${ }^{2} \quad 4 \cos ^{2} x+1=0$ <br> - $\quad \cos x=\frac{1}{2}$ and $\cos x=-\frac{1}{2}$ <br> - ${ }^{4} \quad x=60,300$ <br> -5 $\mathrm{x}=120,240$ |


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| :---: | :---: | :---: |
| 3(a) <br> (b) | ans: $y=x^{2}+\frac{6}{x}-4$ <br> (4 marks) <br> - ${ }^{1}$ knows to integrate <br> - ${ }^{2}$ integrates <br> - ${ }^{3}$ subs point <br> - ${ }^{4}$ solves for $C$ and states function <br> ans: $p=7$ <br> (1 mark) <br> - ${ }^{1} \quad$ subs point and solves for $p$ | - ${ }^{1} y=\int 2 x-\frac{6}{x^{2}} d x$ <br> - $2 y=x^{2}+\frac{6}{x}+C$ <br> - $3=2^{2}+\frac{6}{2}+C$ <br> - $4=x^{2}+\frac{6}{x}-4$ <br> - ${ }^{1} \quad p=3^{2}+\frac{6}{3}-4=7$ |
| 4(a) <br> (b) <br> (c) <br> (d) | ans: $\mathbf{P}(\mathbf{3}, \mathbf{0})$ <br> (2 marks) <br> - ${ }^{1}$ knows to make function equal to 0 <br> - ${ }^{2} \quad$ solves for $x$ and states cords of P <br> ans: $2 y+3 x=9$ <br> (1 mark) <br> - ${ }^{1}$ subs info into formula for straight line <br> ans: $y-11 x=17$ <br> (4 marks) <br> - ${ }^{1}$ knows to take derivative <br> - ${ }^{2}$ subs to find gradient <br> - ${ }^{3}$ subs to find point of contact <br> -4 subs into straight line formula <br> ans: $Q(-1,6)$ <br> (3 marks) <br> - ${ }^{1}$ knows to use sim. eqs. <br> - ${ }^{2} \quad$ solves for $x$ and $y$ <br> - ${ }^{3}$ states coordinates of Q | - $x^{3}-x^{2}-5 x-3=0$ <br> - ${ }^{2} \quad x=3 ; \mathrm{P}(3,0)$ <br> - ${ }^{1} y=-\frac{3}{2}(x-3)$ <br> - $\frac{d y}{d x}=3 x^{2}-2 x-5$ <br> - $23(-2)^{2}-2(-2)-5=11$ <br> - $3 y=(-2)^{3}-(-2)^{2}-5(-2)-3=-5$ <br> $\bullet^{4} y+5=11(x+2) ; y-11 x=17$ <br> - ${ }^{1}$ evidence <br> -2 $x=-1$ and $y=6$ <br> -3 $\mathrm{Q}(-1,6)$ |


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| :---: | :---: | :---: |
| 5(a) <br> (b) <br> (c) | ans: 143.3gu's <br> (2 marks) <br> - ${ }^{1}$ knows how to calculate answer <br> - ${ }^{2}$ answer <br> ans: 135.8 gu 's <br> (3 marks) <br> - ${ }^{1}$ sets up recurrence relation <br> -2 repeated calculations to answer <br> - ${ }^{3}$ repeated calculations to answer <br> ans: yes since lower limit is 80.8 ( 3 marks) <br> -1 knows to find limit <br> - 2 finds limit <br> - realises lower limit is less than 100 | - ${ }^{1} 0.92^{4} \times 200$ <br> - ${ }^{2} 143 \cdot 3 g u^{\prime} s$ <br> - ${ }^{1} \quad U_{n+1}=0 \cdot 92^{4} U_{n}+32$ <br> $\bullet^{2}$ 175.3[after 4 hours]; 157•6[after 8 hours] <br> - ${ }^{3} 144 \cdot 9$ [after 12 hours]; 135•8[after 16 hours] <br> - $\quad L=\frac{32}{1-0.92^{4}}$ <br> - $2 \quad L=112 \cdot 8$ <br> -3 brightness would fall below 100 since lower limit is $80 \cdot 8$ |
| 6(a) <br> (b) | ans: proof <br> - ${ }^{1}$ cross multiplies and multiplies out <br> - ${ }^{2}$ brings to LHS <br> - ${ }^{3}$ rearranges as required <br> ans: $\quad k=\frac{5}{4}$ <br> (5 marks) <br> - ${ }^{1}$ states condition for equal roots <br> - ${ }^{2}$ states values of $a, b$ and $c$ <br> -3 substitutes into $b^{2}-4 a c$ <br> - ${ }^{4}$ multiplies out and simplifies <br> - ${ }^{5}$ solves for $k$ | - ${ }^{1} k\left(x^{2}+4\right)=x^{2}-2 x+1$ <br> - ${ }^{2} k x^{2}-x^{2}+2 x+4 k-1$ <br> - ${ }^{3}(k-1) x^{2}+2 x+(4 k-1)=0$ <br> - ${ }^{1} b^{2}-4 a c=0$ for equal roots [stated/implied] <br> - $\quad a=(k-1) ; b=2 ; c=(4 k-1)$ <br> - $2^{2}-4(k-1)(4 k-1)=0$ <br> - $420 k-16 k^{2}=0$ <br> - ${ }^{5} \quad k=\frac{5}{4}$ |
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| :---: | :---: | :---: |
| 7(a) <br> (b) | ans: proof <br> - ${ }^{1}$ finds expression for length of shed <br> - 2 finds expression for area of g'house <br> - ${ }^{3}$ simplifies to correct form <br> ans: 15 <br> (5 marks) <br> - ${ }^{1}$ knows to equate derivative to 0 <br> - ${ }^{2}$ prepares to differentiate <br> - ${ }^{3}$ differentiates <br> - ${ }^{4}$ solves for $x$ <br> - 5 justifies answer | - ${ }^{1}$ length of shed $=\frac{3}{x}$ <br> - $2 \quad A=(x+3)\left(4+\frac{3}{x}\right)-3$ <br> - $A=4 x+3+12+\frac{9}{x}-3 \rightarrow$ answer <br> - $\frac{d y}{d x}=0$ <br> - $2 A(x)=12+4 x+9 x^{-1}$ <br> - $A^{\prime}(x)=4-\frac{9}{x^{2}}=0$ <br> - $4-\frac{9}{x^{2}}=0 ; x^{2}=\frac{9}{4} ; x=\frac{3}{2}$ <br> ${ }^{5} \downarrow \longrightarrow$ or other acceptable method |
| 8(a) | ans: $\quad \frac{\pi}{2}$ radians <br> (6 marks) <br> - ${ }^{1}$ identifies coefficients <br> - ${ }^{2}$ identifies one factor <br> -3 correct second factor <br> $-{ }^{4}$ <br> $\bullet{ }^{5}$ solves <br> $\bullet$ valid conclusion for $\sin ^{2} \theta=-4$ | $\begin{array}{llllll}{ }^{1} & 1 & -1 & 4 & -4\end{array}$ <br> - ${ }^{2}(\sin \theta-1)$ <br> $\left(\sin ^{2} \theta+4\right)$ <br> $\sin \theta=1$ and $\sin ^{2} \theta=-4$ <br> - ${ }^{5} \theta=\frac{\pi}{2}$ radians <br> - ${ }^{6} \sin 2 \theta=-4$ has no solution |

