Higher Grade Paper 2 2007/2008

Marking Scheme

Give 1 mark for each •	Illustration(s) for awarding each mark
ans: $2y - x = -2$ (3 marks)•1 finds midpoint of BC•2 establishes gradient of AM•3 substitutes in general equation	• midpoint BC: (10,4) • $m_{AM} = \frac{4+2}{10+2} = \frac{1}{2}$ • $y - 4 = \frac{1}{2}(x - 10)$
ans: $D(4,1)$ (3 marks)•1realising $y = 1$ •2substitutes into equation•3states coordinates of D	• ¹ $y = 1$ • ² $2(1) - x = -2; x = 4$ • ³ $D(4,1)$
ans:proof(3 marks)•1finds gradient of BD•2knows condition for perp. lines•3makes statement re perpendicular	• $m_{BD} = -2$ • $m_1 \times m_2 = -1$ [stated or implied] • $\frac{1}{2} \times -2 = -1$ so AM and BD are perp.
ans: $P(1,-\frac{25}{2})$ (4 marks) • ¹ knows to take derivative and equate to 0 • ² takes derivative • ³ solves to find x - coordinate • ⁴ substitutes to find y - coordinate ans: $Q(6,0)$ (3 marks) • ¹ knows to make $y = 0$ • ² uses synthetic division to find x • ³ states coordinates of Q	• $\frac{dy}{dx} = 0$ • $\frac{dy}{dx} = 0$ • $\frac{dy}{dx} = 15x + 12 = 0$ • $\frac{dy}{dx} = 1 \text{ [or 4]}$ • $\frac{dy}{dx} = 1 \text{ [or 4]}$ • $\frac{dy}{dx} = 13 - \frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$ • $\frac{dy}{dx} = 13 - \frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$ • $\frac{dy}{dx} = -\frac$
	Give 1 mark for each •ans: $2y - x = -2$ (3 marks)•1finds midpoint of BC•2establishes gradient of AM•3substitutes in general equationans: $D(4,1)$ (3 marks)•1realising $y = 1$ •2substitutes into equation•3states coordinates of Dans: proof(3 marks)•1finds gradient of BD•2knows condition for perp. lines•3makes statement re perpendicularans: $P(1, -\frac{25}{2})$ (4 marks)•1knows to take derivative and equate to 0•2takes derivative•3solves to find x - coordinate•4substitutes to find y - coordinateans: $Q(6,0)$ (3 marks)•1knows to make $y = 0$ •2uses synthetic division to find x•3states coordinates of Q

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	ans:32.6 gigatonnes(3 marks)•1correct multiplier•2completes calculation•3calculation and correct rounding	• 1 0.96 • 2 0.96 ⁵ × 40 • 3 32.6 gigatonnes
(b)	ans: 31 gigatonnes (3 marks)	
	 ¹ sets up recurrence relation ² knows to calculate 3 figures ³ final answer 	• $U_{n+1} = 0.96^5 U_n + 3.8$ • 1^{st} year: 36.4; 2^{nd} year: 33.4795 • 3^{rd} year: 31 gigatonnes
(c)	ans: upper 20.6; lower 16.8 (3 marks)	
	 ¹ knows limit exists ² finds upper limit ³ finds lower limit 	• limit exists since $-1 < 0.96^5 < 1$ • $L = \frac{3 \cdot 8}{1 - (0.96)^5} = 20.6$ • $20.6 - 3.8 = 16.8$
4(a)	ans: $a = -2$ (2 marks)	
	 finds expression for f(g(-2)) equates to -1 and solves for a 	• ¹ $f(g(-2)) = f(-1) = 1 + a$ • ² $a = -2$
(b)	ans: $x = -2, 0, 2$ (5 marks)	
	 ¹ substitutes ² simplifies ³ equates to 2 ⁴ factorises ⁵ solves for x 	• ¹ $f(f(x)) = (x^2 - 2)^2 - 2$ • ² $x^4 - 4x^2 + 2$ • ³ $x^4 - 4x^2 + 2 = 2; x^4 - 4x^2 = 0$ • ⁴ $x^2(x^2 - 4) = 0$ • ⁵ $x = -2, 0, 2$
5(a)	ans: $x = 1$ (2 marks)	
	• ¹ realises $y = 0$; equates to 0 • ² solves for x	• ¹ $3x^2 - 6x + 3 = 0$ • ² $3(x - 1)^2 = 0; x = 1$
(b)	ans: $b = 2$ (5 marks)	
	 integrates expression substitutes values simplifies, equates to 1, rearranges uses synthetic division to solve realises one solution; discards b² - b + 1 	• ¹ $[x^{3} - 3x^{2} + 3x]_{1}^{b}$ • ² $(b^{3} - 3b^{2} + 3b) - (1 - 3 + 3)$ • ³ $b^{3} - 3b^{2} + 3b - 2 = 0$ • ⁴ $2 \begin{bmatrix} 1 & -3 & 3 & -2 \\ 2 & -2 & 2 \\ 1 & -1 & 1 & 0 \end{bmatrix}$ • ⁵ $b = 2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(c)	ans: $y = 6x - 9$ (4 marks)	
	 finds 'c' knows to differentiate substitutes to find gradient substitutes into general equation 	• when $x = 2, y = 3$; Q(2,3) • $\frac{dy}{dx} = 6x - 6$ • when $x = 2$; $\frac{dy}{dx} = 6$ • $y - 3 = 6(x - 2)$
6(a)	ans: (-2,1) (3 marks)	
	 ¹ substitutes eq.of line in eq. of circle ² simplifies and solves for x ³ substitutes to find y 	• ¹ $x^{2} + (2x+5)^{2} - 4x + 2(2x+5) - 15 = 0$ • ² $5(x+2)^{2} = 0; x = -2$ • ³ $y = 2(-2) + 5; y = 1$
(b)	ans: $(x+6)^2 + (y-3)^2 = 20$ (3 marks)	
	 •¹ establishes coordinates of B •² finds r² •³ substitutes into general circle equation 	• B(-6,3) • $r^2 = 20$ • $(x+6)^2 + (y-3)^2 = 20$
7(a)	ans: $BE = 3$ units (2 marks)	
	\bullet^1 uses $\tan \frac{\pi}{2}$	\bullet^1 evidence
	• ² finds length	\bullet^2 BE = 3
(b)	ans: proof (4 marks)	
	• ¹ finds length BC	\bullet^1 $\frac{5}{\sqrt{2}}$
	\bullet^2 Finds area of triangle AEB	$\bullet^2 \frac{9\sqrt{3}}{2}$ units ²
	\bullet^3 states length BD and finds area of triangle	2
	BCD	• ³ BD = BC; area = $\frac{25}{4}$ units ²
	⁴ completes simplification to answer	• ⁴ $\frac{1}{4}(18\sqrt{3}+25)$ units ²
8(a)	ans: $m = 2400 \text{ml}$ (4 marks)	
	• ¹ knows to differentiate and equate to 0	• ¹ $H'(m) = 0$
	• ² differentiates	• ² $4 - \frac{1}{600}m = 0$
	 ³ solves for x ⁴ justifies maximum 	• ³ $m = 2400$ • ⁴ table of values; second derivative
(b)	ans: 4800 feet (1 mark)	
	• ¹ knows to sub into function and evaluate	• 1 4(2400) $-\frac{(2400)^{2}}{1200} = 4800$ feet

Total: 60 marks