Further Calculus

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Differentiate $2(4-x)^{-\frac{1}{2}}$ with respect to x.
   A. $(4-x)^{-1}$
   B. $-(4-x)^{-1}$
   C. $(4-x)^{-\frac{3}{2}}$
   D. $-(4-x)^{-\frac{3}{2}}$

2. What is the gradient of the tangent to the curve with equation $y = \cos 2x$ at the point where $x = \frac{\pi}{4}$?
   A. -2
   B. -1
   C. 0
   D. 2

3. Find $\int (2x^{-4} + \cos 5x)\,dx$.
   A. $-\frac{2}{5}x^{-5} - 5\sin 5x + c$
   B. $-\frac{2}{5}x^{-5} + \frac{1}{5}\sin 5x + c$
   C. $-\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + c$
   D. $-\frac{2}{3}x^{-3} - 5\sin 5x + c$

4. Differentiate $3\cos\left(2x - \frac{\pi}{6}\right)$ with respect to x.
   A. $-3\sin(2x)$
   B. $-3\sin(2x - \frac{\pi}{6})$
   C. $-6\sin(2x - \frac{\pi}{6})$
   D. $6\sin(2x - \frac{\pi}{6})$

5. Given that $f(x) = 3\cos(2x)$, what is the value of $f'\left(\frac{\pi}{6}\right)$?
   A. 3
   B. $-3\sqrt{3}$
   C. -3
   D. $\frac{3\sqrt{3}}{2}$

6. Given that $f(x) = 4\sin 3x$, find $f'(0)$.
   A. 0
   B. 1
   C. 12
   D. 36

7. Given that $f(x) = \frac{1}{2}\sin^2 x$, what is the value of $f'\left(\frac{\pi}{3}\right)$?
   A. $-\frac{1}{2}$
   B. $\sqrt{3}$
   C. $\frac{\sqrt{3}}{2}$
   D. $\frac{\sqrt{3}}{4}$

8. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.

   A. $-3x(4 - 3x^2)^{-\frac{1}{2}}$

   B. $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$

   C. $2(4 - 3x^3)^{\frac{1}{2}}$

   D. $3x(4 - 3x^2)^{-\frac{3}{2}}$

9. Differentiate $(6x^2)^5$ with respect to x.

   A. $60x^9$

   B. $5(6x^2)^4$

   C. $30(6x^2)^4$

   D. $60x(6x^2)^4$

10. A function f is defined for $x \leq 4$ by $f(x) = (8 - 2x)^{\frac{3}{2}}$.

    What is the value of $f'(2)$?

    A. -24

    B. -6

    C. 3

    D. 8

11. If $y = 3\cos^4 x$, find $\frac{dy}{dx}$.

    A. $12\cos^3 x \sin x$

    B. $12\cos^3 x$

    C. $-12\cos^3 x \sin x$

    D. $-12\sin^3 x$

12. Find $\int (2x - 1)^{\frac{1}{2}}\, dx$ where $x > \frac{1}{2}$.

    A. $\frac{1}{3}(2x - 1)^{\frac{3}{2}} + c$

    B. $\frac{1}{2}(2x - 1)^{-\frac{1}{2}} + c$

    C. $\frac{1}{2}(2x - 1)^{\frac{3}{2}} + c$

    D. $\frac{1}{3}(2x - 1)^{-\frac{1}{2}} + c$

13. Find $\int (2x - 5)^4\, dx$.

    A. $8(2x - 5)^3 + c$

    B. $4(2x - 5)^3 + c$

    C. $\frac{1}{5}(2x - 5)^5 + c$

    D. $\frac{1}{10}(2x - 5)^5 + c$

14. What is the value of $\int_0^{\pi} \sin x\, dx$?

    A. -2

    B. 0

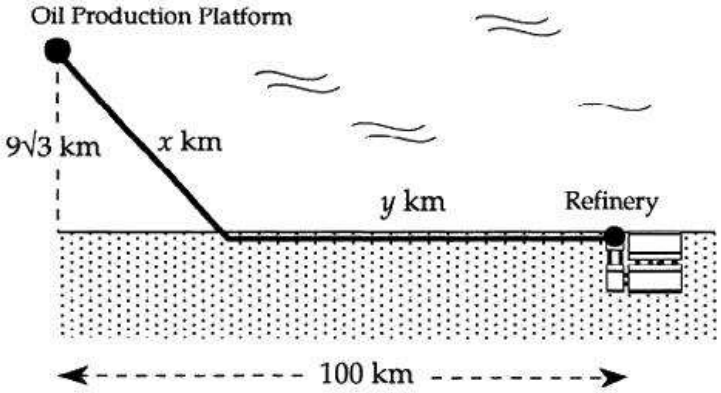
    C. 1

    D. 2

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions

[SQA] 15. An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(*a*) Show that the total cost of this pipeline is £$C(x)$ million where

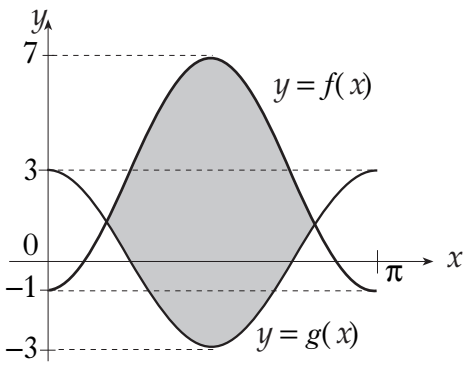
$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$ **(3)**

(*b*) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

[SQA] 16. Find the equation of the tangent to the curve $y = 2\sin(x - \frac{\pi}{6})$ at the point where $x = \frac{\pi}{3}$. **4**

[SQA] 17. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.



(*a*) Write down the values of m and n. **1**

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$. **5**

(*c*) Calculate the shaded area. **6**

[SQA] 18. A point moves in a straight line such that its acceleration a is given by $a = 2(4-t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

[SQA] 19. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.
If $f'(x) = \sin(3x)$ express y in terms of x. **4**

[SQA] 20. A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5\pi}{12}, \sqrt{3}\right)$.
Find y in terms of x. **4**

[SQA] 21. Differentiate $\sin 2x + \frac{2}{\sqrt{x}}$ with respect to x. **4**

[SQA] 22. Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

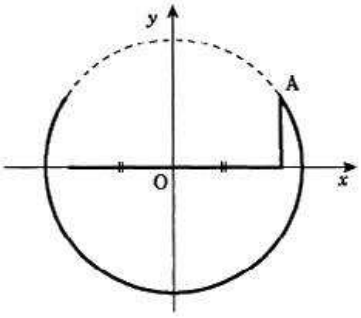
[SQA] 23. Given $f(x) = \cos^2 x - \sin^2 x$, find $f'(x)$. **3**

[SQA] 24. Given that $f(x) = 5(7-2x)^3$, find the value of $f'(4)$. **4**

[SQA] 25. Differentiate $2x^{\frac{3}{2}} + \sin^2 x$ with respect to x. **4**

[SQA] 26. Find the derivative, with respect to x, of $\frac{1}{x^3} + \cos 3x$. **4**

[SQA] 27. Linktown Church is considering designs for a logo for their Parish magazine. The 'C' is part of a circle and the centre of the circle is the mid-point of the vertical arm of the 'L'. Since the 'L' is clearly smaller than the 'C', the designer wishes to ensure that the total length of the arms of the 'L' is as long as possible.



The designer decides to call the point where the 'L' and 'C' meet A and chooses to draw co-ordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is $x^2 + y^2 = 20$.

(a) If A has co-ordinates (x,y), show that the total length T of the arms of the 'L' is given by $T = 2x + \sqrt{20 - x^2}$. **(1)**

(b) Show that for a stationary value of T, x satisfies the equation

$$x = 2\sqrt{20 - x^2}.$$ **(5)**

(c) By squaring both sides, solve this equation.
Hence find the greatest length of the arms of the 'L'. **(3)**

[SQA] 28. If $f(x) = \cos^2 x - \dfrac{2}{3x^2}$, find $f'(x)$. **4**

[SQA] 29. Differentiate $4\sqrt{x} + 3\cos 2x$ with respect to x. **4**

[SQA] 30. Differentiate $\sin^3 x$ with respect to x.

Hence find $\int \sin^2 x \cos x \, dx$. **4**

[SQA] 31. Find $\dfrac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$. **3**

[SQA] 32. Given $f(x) = (\sin x + 1)^2$, find the exact value of $f'(\frac{\pi}{6})$. **3**

33. (*a*) A curve has equation $y = (2x - 9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

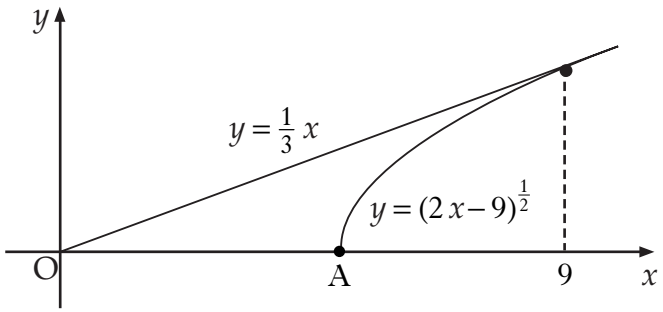


Diagram 1

Find the coordinates of point A. **1**

(*c*) Calculate the shaded area shown in diagram 2. **7**

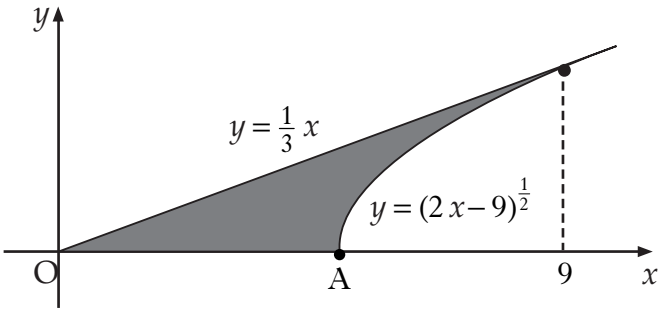


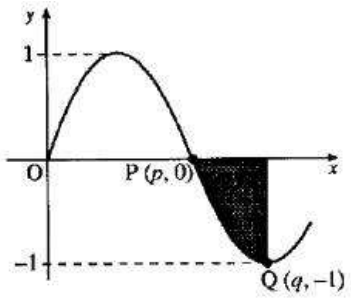
Diagram 2

[SQA] 34. Find $\int \sqrt{1+3x}\,dx$ and hence find the exact value of $\int_0^1 \sqrt{1+3x}\,dx$. **4**

[SQA] 35. Find $\int \frac{1}{(7-3x)^2}\,dx$. **2**

[SQA] 36. Evaluate $\int_{-3}^{0} \left(2x+3\right)^2 dx$. **4**

[SQA] 37. A sketch of part of the graph of $y = \sin 2x$ is shown in the diagram.
The points P and Q have coordinates $(p, 0)$ and $(q, -1)$.



(*a*) Write down the values of p and q. **1**

(*b*) Find the area of the shaded region. **4**

[SQA] 38.

(*a*) Evaluate $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$. **3**

(*b*) Draw a sketch and explain your answer. **2**

[SQA] 39.

(*a*) Show that $(\cos x + \sin x)^2 = 1 + \sin 2x$. **1**

(*b*) Hence find $\int (\cos x + \sin x)^2 \, dx$. **3**

[SQA] 40. Find $\int \left(6x^2 - x + \cos x\right) dx$. **4**

[SQA] 41. The curve $y = f(x)$ passes through the point $(\frac{\pi}{12}, 1)$ and $f'(x) = \cos 2x$.
Find $f(x)$. **3**

[SQA] 42.

(*a*) By writing $\sin 3x$ as $\sin(2x + x)$, show that $\sin 3x = 3 \sin x - 4 \sin^3 x$. **4**

(*b*) Hence find $\int \sin^3 x \, dx$. **4**

[SQA] 43. A function f is **EVEN** if $f(-x) = f(x)$

e.g. when $f(x) = x^2$, f is **EVEN** because $f(-x) = (-x)^2 = x^2 = f(x)$.

A function f is **ODD** if $f(-x) = -f(x)$

e.g. when $f(x) = x^3$, f is **ODD** because $f(-x) = (-x)^3 = -x^3 = -f(x)$.

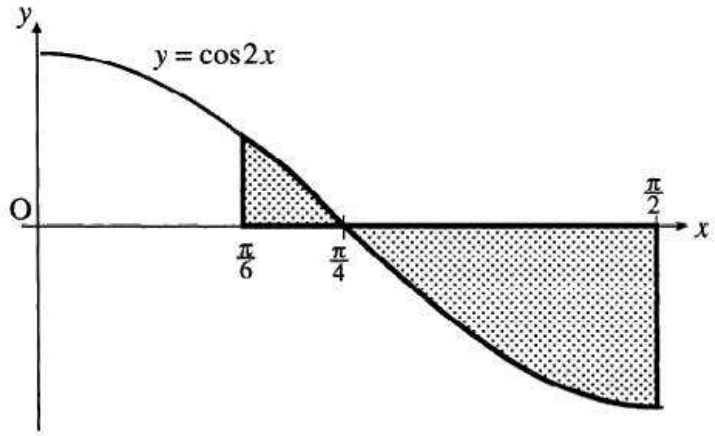
(*a*) Given that $g(x) = \cos x$ and $h(x) = \sin 2x$, decide for each of the functions g and h whether it is **EVEN** or **ODD**.
Justify your decisions. **(4)**

(*b*) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \ dx$ and $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x \ dx$. **(5)**

(*c*) On separate diagrams, draw rough sketches of the graphs of $y = \cos x$ and $y = \sin 2x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. **(2)**

(*d*) If $v(x) = x\cos x$, check whether the function v is **EVEN** or **ODD** and suggest a value for $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x \ dx$. **(2)**

[SQA] 44. **An artist has designed a 'bow' shape which he finds can be modelled by the shaded area below. Calculate the area of this shape.** **(6)**



[SQA] 45. (*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. **2**

(*b*) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} \, dx$. **1**

46. (*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R > 0$ and $0 \leq a < 2\pi$.

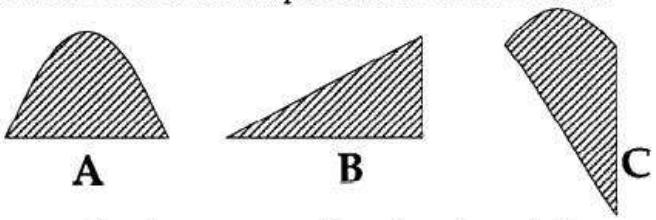
Calculate the values of R and a. **4**

(*b*) Hence find the value of t, where $0 \leq t \leq 2$, for which

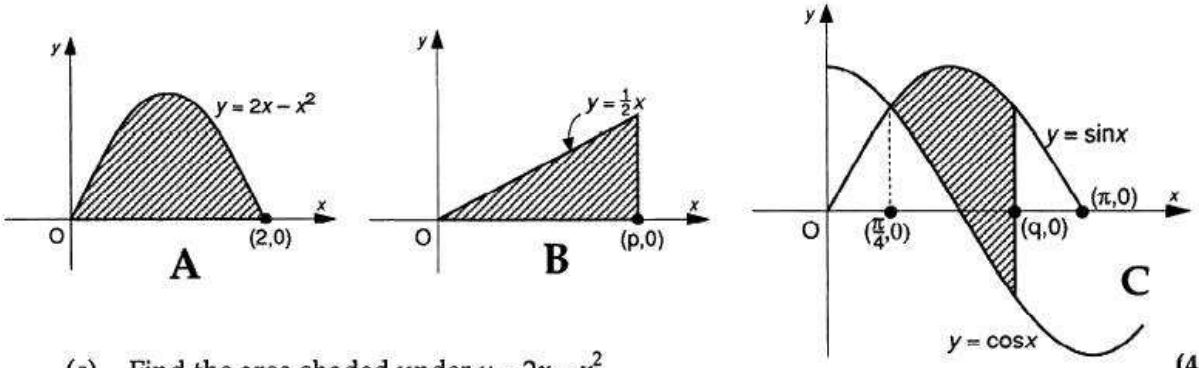
$$\int_0^t (3\cos x + 5\sin x)\, dx = 3.$$

7

[SQA] 47. An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the **same** area. Three of the shapes used are drawn below.



Relative to x,y–axes, the shapes are positioned as shown below.



(*a*) Find the area shaded under $y = 2x - x^2$. **(4)**

(*b*) Use the area found in part (*a*) to find the value of p. **(2)**

(c) Prove that q satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of q to 2 significant figures. **(10)**

[END OF WRITTEN QUESTIONS]