Higher Grade Paper 2 2011/2012

Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a) (b)	ans: $3y - x = 4$ \bullet^1 finds gradient \bullet^2 finds equation of altitudeans: $x = 4$ \bullet^1 states equation of median	• ¹ m_{BC} = -3 therefore m(altitude) = $\frac{1}{3}$ • ² $y - 2 = \frac{1}{3}(x - 2)$ • ¹ from mid-point (4,3) states x=4
(c)	ans: $(4, \frac{8}{3})$ • ¹ subs x=4 into altitude to find y • ² states coords of intersection	• $y = \frac{8}{3}$ • $(4, \frac{8}{3})$
2(a)	 ans: proof (3 marks) •¹ subs one function into the other •² multiplies inner bracket •³ multiplies to answer 	• $f(x-3) = (x-3-1)^2 = (x-4)^2$ • $h(x) = [x^2 - 8x + 16]x^2$ • $x^4 - 8x^3 + 16$
(b)	ans: A(2, 16) (5 marks) • ¹ knows to make $\frac{dy}{dx} = 0$ • ² differentiates • ³ solves for x • ⁴ chooses correct values & subs to find y • ⁵ states coordinates of A	• $\frac{dy}{dx} = 0$ • $\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0$ at SP • $\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0$ at SP • $\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0$ at SP • $\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0$ at SP • $\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0$ at SP • $\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0$ at SP
3(a)	ans: $p = 0.5$ (4 marks)•1 gives expression for both limits•2 equates limits•3 starts to solve•4 solves and discards	• ¹ $L = \frac{6}{1-p}; L = \frac{9}{1-p^2}$ • ² $\frac{6}{1-p} = \frac{9}{1-p^2}$ • ³ $6-6p^2 = 9-9p; 6p^2 - 9p + 3 = 0$ • ⁴ $3(2p-1)(p-1) = 0; p = 0.5 \text{ or } p = 1$
(b)	 ans: 22 (3 marks) ¹ finds 1st term for one RR ² finds 1st term for other RR 	• $U_1 = \frac{1}{2}(100) + 6 = 56$ • $U_1 = (\frac{1}{2})^2(100) + 6 = 34$

\bullet^3 calculates difference in terms

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4	ans: $y = x^3 + x^2 + 3$ (4 marks) • ¹ knows to integrate • ² answer • ³ subs for x and y to solve for c • ⁴ Equation	• ¹ $y = x^{3} + x^{2}$ • ² $y = x^{3} + x^{2} + c$ • ³ $c = 3$ • ⁴ $y = x^{3} + x^{2} + 3$
5(a)	ans: $P(-2, 0)$ (3 marks)•1equates function to 0•2solves using suitable strategy•3states coordinates of P	• $x^{3} + 6x^{2} + 12x + 8 = 0$ at P • suitable strategy leading to $x = -2$ • P(-2, 0)
(b)	ans: 4 square units (4 marks)	
	\bullet^1 knows how to find area	• $\int_{-2}^{0} x^3 + 6x^2 + 12x + 8 dx$
	\bullet^2 integrates	• ² $\left[\frac{x^4}{4} + 2x^3 + 6x^2 + 8x\right]_{-2}^{0}$
	\bullet^3 subs values	• ³ $0 - \left(\frac{(-2)^4}{4} + 2(-2)^3 + 6(-2)^2 + 8(-2)\right)$
	\bullet^4 evaluates	• ⁴ 4 square units
6(a)	ans: $y + x = 3$ (3 marks)	
	• ¹ finds gradient of CP	• ¹ $m_{tangent} = -1$ so $m_{radius} = 1$
	\bullet^2 Subs into straight line equation	• ² $y - 7 = -1(x + 4)$
(b)	• ¹ Subs (2 ,k) into CP	• ¹ $k = 1$
(c)	ans: $(x-2)^2 + (y-1)^2 = 18$ (3 marks)	
	 ¹ finds midpoint of CP ² finds radius (length of CQ) ³ subs into general equation of circle 	• ¹ Q(-1, 4) • ² $r^2 = 3^2 + 3^2 = 18$ • ³ $(x-2)^2 + (y-1)^2 = 18$

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7(a)	 ans: proof (3 marks) ¹ gives expression for length and breadth ² subs into formula and starts to simplify ³ completes simplification to answer 	• ¹ (30-2x) • ² $x(30-2x)^2$ • ³ $x(900-120x+4x^2)$
(b)	ans: $x = 5$ (5 marks) • ¹ knows to make derivative = 0 • ² takes derivative • ³ factorises and solves • ⁴ discards • ⁵ justifies answer	• $V'(x) = 0$ • $12x^2 - 240x + 900 = 0$ • $12(x-5)(x-15) = 0$ • $x = 5$ • nature table or 2^{nd} derivative
(c)	ans: 2 litres(1 mark) \bullet^1 calculates volume	• ¹ $20 \times 20 \times 5 = 2000 \text{ cm}^3 = 2$ litres
8	ans: $\frac{\pi}{6}, \frac{5\pi}{2}, \frac{3\pi}{2}$ radians (5 marks) • ¹ factorises • ² begins to solve • ³ solves sinx = $\frac{1}{2}$ • ⁴ • ⁵ solves sinx = -1	• ¹ $(2\sin x - 1)(\sin x + 1) = 0$ • ² $2\sin x = 1$ and $\sin x = -1$ • ³ $x = \frac{\pi}{6}$ radians • ⁴ $x = \frac{5\pi}{6}$ radians • ⁵ $x = \frac{3\pi}{2}$ radians

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9(a)	ans: proof (3 marks)	
	 •¹ states expression for both distances •² equates •³ rearranges to answer 	• $\frac{3}{k-x}$ and $\frac{4x}{k}$ • $\frac{3}{k-x} = \frac{4x}{k}$ • $\frac{3}{k-x} = \frac{4x}{k}$ • $\frac{3}{k} = 4x(k-x); 3k = 4xk - 4x^2$
(b)	ans: $k = 3$ (3 marks)	
	 ¹ knows discriminant = 0 for equal roots ² finds discriminant ³ solves and discards 	• $b^{2} - 4ac = 0$ for equal roots • $b^{2} - 4ac = (-4k)^{2} - 4.4.3k = 0;16k^{2} - 48k = 0$ • $16k(k-3) = 0; k = 3$
(c)	ans: $x = \frac{3}{2}$ (2 marks)	
	 ¹ subs value for k and rewrites expression ² factorises and solves 	• ¹ $4x^2 - 12x + 9 = 0$ • ² $(2x - 3)^2 = 0; \ x = \frac{3}{2}$
		Total: 59 marks