



DINGWALL ACADEMY

Prelim Examination 2011 / 12

Mathematics
Higher Prelim Examination 2011/2012
Paper 2
Assessing Units 1 & 2

NATIONAL
QUALIFICATIONS

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

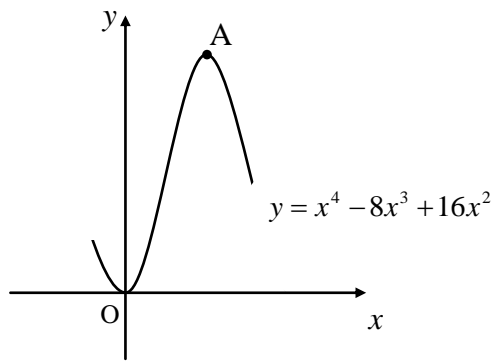
$$= 1 - 2 \sin^2 A$$

ALL questions should be attempted

1. A triangle has vertices A(2, 2), B(4, 10) and C(6, 4).
- (a) Find the equation of the Altitude through point A. 5
 - (b) Find the equation of the Median through B
 - (c) Hence find the coordinates of the intersection G, of the median and the altitude. 5

2. (a) A function f , defined on a suitable domain, is given as $f(x) = (x - 1)^2$.
- A second function h is such that $h(x) = [f(x - 3)]x^2$.
- Show clearly that h can be written in the form $h(x) = x^4 - 8x^3 + 16x^2$. 3

- (b) Part of the graph of $y = h(x)$ is shown below.



- Find the coordinates of point A. 5

3. Two unique sequences are defined by the following recurrence relations

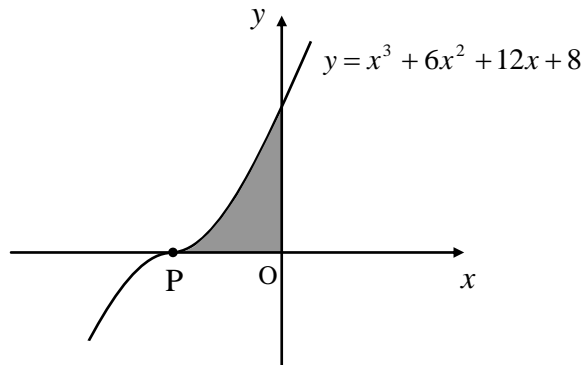
$$U_{n+1} = pU_n + 6 \quad \text{and} \quad U_{n+1} = p^2U_n + 9, \quad \text{where } p \text{ is a constant.}$$

- (a) If both sequences have the same limit, find the value of p . 4
- (b) For both sequences the value of $U_0 = 100$. Find the difference between the value of the U_1 terms for each sequence. 3

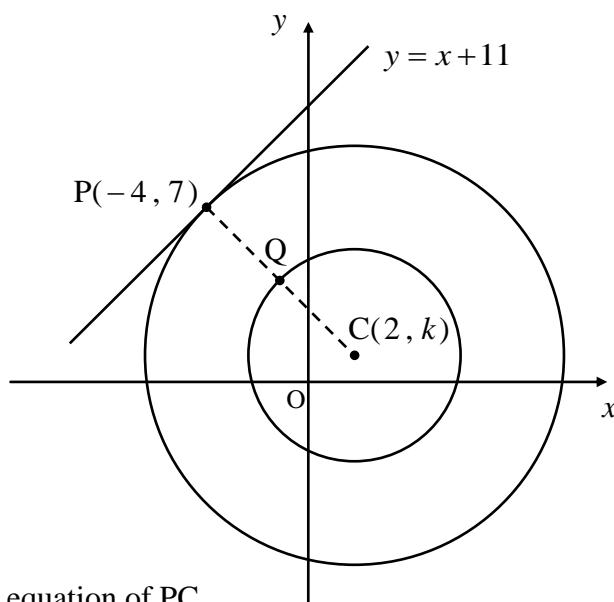
4. Find the equation of the function $y = f(x)$ for which $\frac{dy}{dx} = 3x^2 + 2x$,
passing through the point (2,15).

4

5. Part of the graph of $y = x^3 + 6x^2 + 12x + 8$ is shown in the diagram.



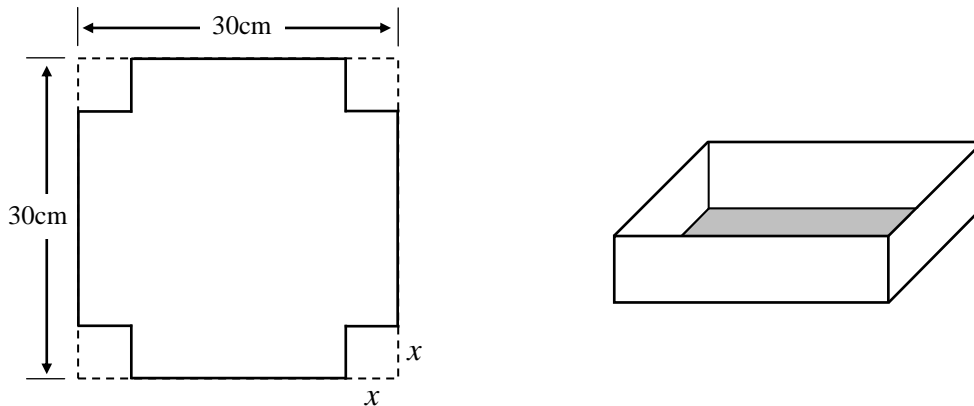
- (a) Find the coordinates of P. 3
- (b) Hence calculate the shaded area. 4
6. The diagram shows two **concentric** circles with centre $C(2, k)$.
The larger of the two circles has the line with equation $y = x + 11$ as a tangent.
The point $P(-4, 7)$ is the point of tangency between this line and the circle.



- (a) Find the equation of PC 2
- (b) Hence find the value of k , the y -coordinate of the point C. 1
- (c) Hence find the equation of the smaller circle given that Q is the mid-point of PC. 3

7. From a square sheet of metal of side 30 centimetres, equal squares of side x centimetres are removed from each corner.

The sides are then folded up and sealed to form an open cuboid.



- (a) Show that the volume of this resulting cuboid is given by

$$V(x) = 4x^3 - 120x^2 + 900x. \quad 3$$

- (b) If the cuboid is to have **maximum** possible volume, what size of square should be removed from each corner? 5

- (c) How many litres of water would this particular cuboid hold? 1

8. Solve the equation $2\sin^2x + \sin x - 1 = 0$, where $0 \leq x \leq 2\pi$ radians 5

9. A designer is testing two model racing cars along a straight track.

Each car completes a single run and the following information is recorded.

	Speed	Distance
Car A	$k - x$	3
Car B	k	$4x$



- (a) Given that both cars completed the run in **exactly the same time**, show clearly that the following equation can be constructed.

$$4x^2 - 4kx + 3k = 0 \quad 3$$

- (b) Find the value of the constant k if the equation $4x^2 - 4kx + 3k = 0$ has **equal roots** and $k > 0$. 3

- (c) Hence find x when k takes this value. 2