



DINGWALL ACADEMY

Prelim Examination 2011 / 12

Mathematics
Higher Prelim Examination 2011/2012
Paper 1
Assessing Units 1 & 2

**NATIONAL
QUALIFICATIONS**

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

- 1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2011/2012 (Section A)**.
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- 3 Make sure you write your **name, class and teacher** on the answer sheet provided.
- 4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
- 5 There is **only one correct** answer to each question.
- 6 Rough working should **not** be done on your answer sheet.
- 7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

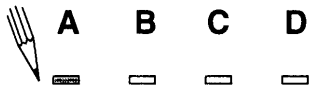
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k, 7)$ lies on this line, the value of k is

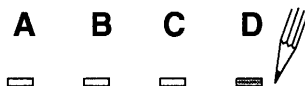
- A 2
- B 27
- C 1.5
- D -2

The correct answer is **A** → 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

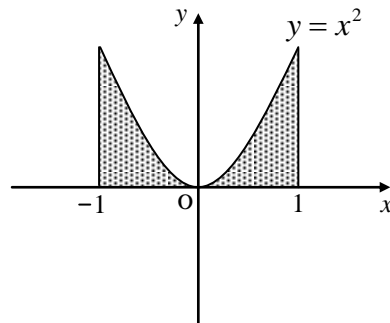
SECTION A

ALL questions should be attempted

1. The gradient of any line perpendicular to the line with equation $3x + 2y = 5$ is
- A -3
 - B $\frac{2}{3}$
 - C $-\frac{3}{2}$
 - D $\frac{1}{3}$
2. The rate of change of the function $y = x^3$ when $x = -1$ is
- A -1
 - B 0
 - C 1
 - D 3
3. A sequence is defined by the recurrence relation $U_{n+1} = 0.5U_n + 12$ with $U_0 = 16$.
 $U_1 - U_2$ equals
- A 42
 - B -2
 - C 4
 - D 2

4. The shaded area in the diagram equals

- A $\frac{1}{3}$ square units
- B 4 square units
- C $\frac{2}{3}$ square units
- D 1 square unit



5. Two functions, defined on suitable domains, are given as $f(x) = \frac{1}{x} - 4$ and $g(x) = -8x$.

The value of $g(f(0.5))$ is

- A $-4\frac{1}{4}$
- B -8
- C 16
- D -16

6. The equation of a circle with radius 4 and centre (3,7) is

- A $(x + 3)^2 + (y + 7)^2 = 16$
- B $(x - 3)^2 + (y - 7)^2 = 4$
- C $(x + 3)^2 + (y + 7)^2 = 4$
- D $(x - 3)^2 + (y - 7)^2 = 16$

7. If $\sin \theta = \frac{1}{\sqrt{5}}$, $\tan \theta =$

- A $\frac{2}{\sqrt{5}}$
- B 2
- C $\frac{1}{2}$
- D $\sqrt{5}$

8. The remainder on dividing the polynomial $x^3 - 3x + 6$ by $x - 2$ is

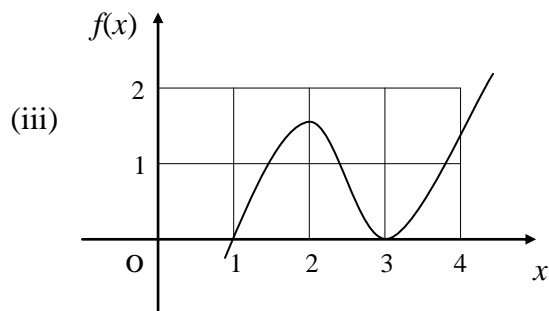
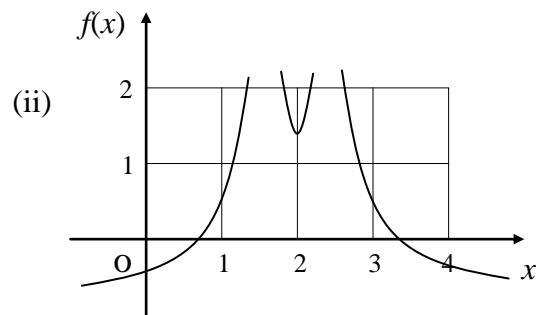
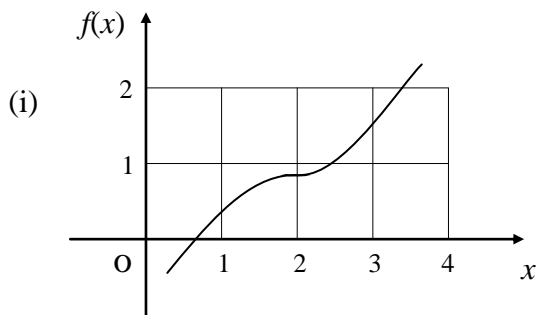
- A 4
- B 8
- C 16
- D 6

9. The function f such that $f(x) = (x-1)(x+5)$ has a stationary value when x equals

- A -5
- B 2
- C -2
- D 1

10. Which of the graphs (i), (ii) or (iii) could be that of a function f such that

$$f'(1) > 0, \quad f'(2) = 0 \quad \text{and} \quad f'(3) > 0?$$



- A (i) only
- B (ii) only
- C (iii) only
- D (i) and (iii) only

11. All the values of x which satisfy $x^2 - x - 12 \geq 0$ are

- A $-4 \leq x \leq 3$
- B $-3 \leq x \leq 4$
- C $x < -3$ or $x > 4$
- D $x \leq -3$ or $x \geq 4$

12. With k being the constant of integration, $\int x^{\frac{1}{2}} dx$ equals

A $\frac{3}{2}x^{\frac{3}{2}} + c$

B $\frac{1}{2x^{\frac{1}{2}}} + c$

C $\frac{1}{2}x^{\frac{3}{2}} + c$

D $\frac{2}{3}x^{\frac{3}{2}} + c$

13. Given that the points $(-2, 1)$, $(0, 7)$ and $(1, k)$ are collinear, then k equals

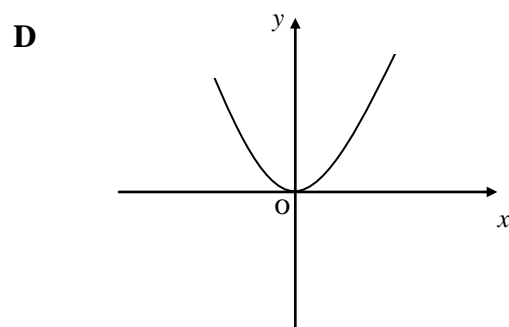
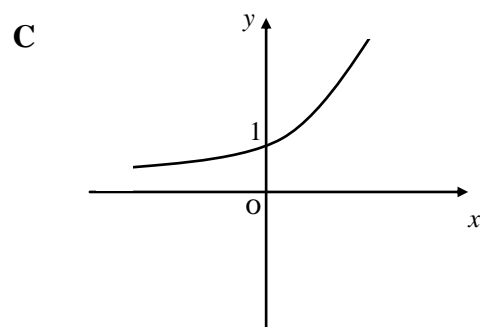
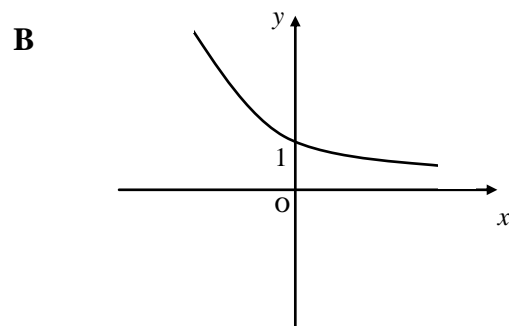
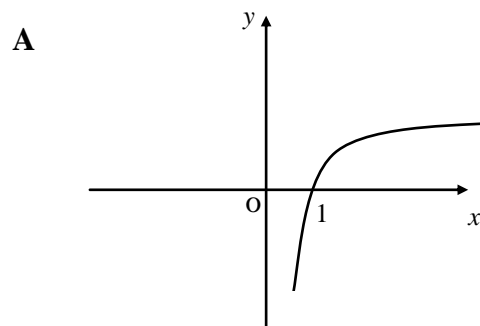
A 13

B 10

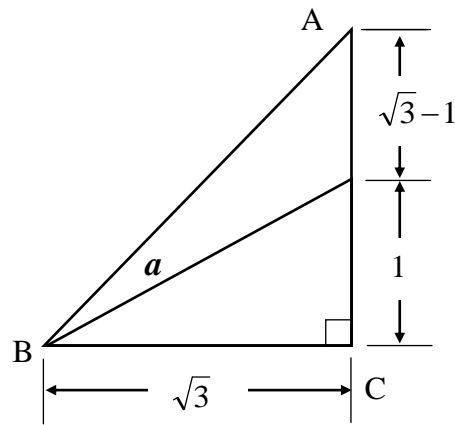
C 0

D -18

14. Which of the following could represent part of the graph of $y = 2^x$?



15. Consider the diagram



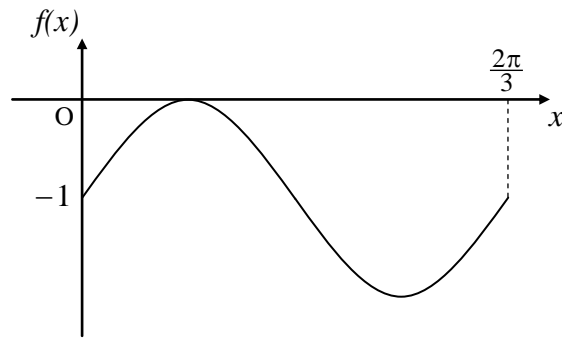
If angle $ABC = \frac{\pi}{4}$, radians then angle a , in radians, is

- A** $\frac{\pi}{6}$
- B** $\frac{\pi}{12}$
- C** $\frac{\pi}{4}$
- D** unknown without the use of a calculator
16. Here are 4 terms used to describe the roots of a quadratic equation
(1) real (2) unequal (3) equal (4) non-real
- Which of them describe(s) the roots of $2x^2 - 3x + 1 = 0$?
- A** (4) only
- B** (3) only
- C** (1) and (3)
- D** (1) and (2)
17. The circle with equation $x^2 + y^2 = 25$ is moved 6 units to the left parallel to the x -axis and 4 units down parallel to the y -axis.

The equation of the circle in this new position is

- A** $(x-6)^2 + (y+4)^2 = 25$
- B** $(x+6)^2 + (y+4)^2 = 25$
- C** $(x+6)^2 + (y-4)^2 = 25$
- D** $(x-6)^2 + (y-4)^2 = 25$

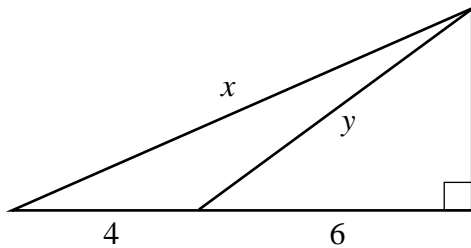
18. The diagram below shows part of the graph of a trigonometrical function.



The most likely function could be $f(x) = \dots$

- A $-\sin x$
- B $-\cos 3x$
- C $\sin 3x - 1$
- D $1 - \sin 3x$

- 19.



From the above diagram, the value of $x^2 - y^2$ is

- A 64
 - B 16
 - C 8
 - D 4
20. Which of the following represents the area between the two curves $y = x^2$ and $y = 8 - x^2$?
- A $\int_{-2}^2 (x^2 - (8 - x^2)) dx$
 - B $\int_{-\sqrt{8}}^{\sqrt{8}} (8 - x^2) - (x^2) dx$
 - C $\int_{-2}^2 (8 - x^2) - (x^2) dx$
 - D $\int_{-\sqrt{8}}^{\sqrt{8}} (x^2) - (8 - x^2) dx$

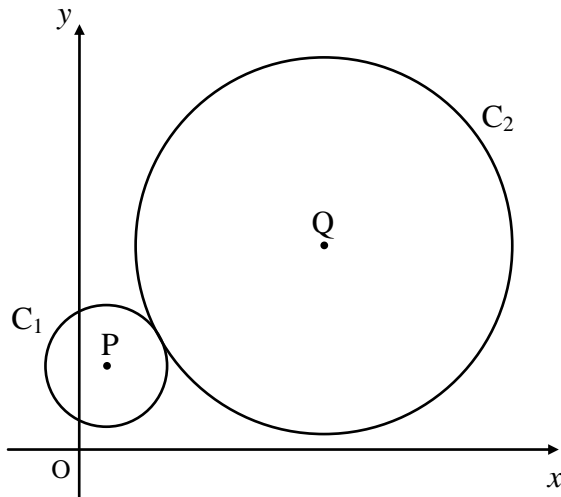
[END OF SECTION A]

SECTION B

ALL questions should be attempted

21. The circle C_1 has $P(1, 3)$ as its centre and a radius of $\sqrt{5}$ units.

The circle C_2 has as its equation $x^2 + y^2 - 18x - 14y + 85 = 0$.



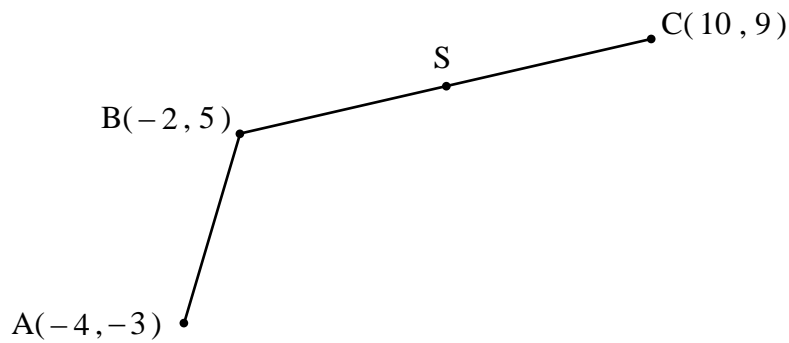
(a) Find the coordinates of Q , the centre of C_2 , and the radius of this larger circle. **3**

(b) Show clearly that C_1 touches C_2 at a single point. **3**

22. Given that $\int_0^a (4 - 3x)^2 dx = 8$, find the value of a . **5**

23. A, B and C have coordinates $(-4, -3)$, $(-2, 5)$ and $(10, 9)$ respectively as shown.

S is the mid-point of BC.



- (a) Find the equation of the line through S parallel to AB. **4**
- (b) Find the coordinates of the point D where ABSD is a parallelogram. **2**
24. Find the equation of the tangent to the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ at the point $(6, 4)$ **4**

25. The line $x = 3y + 10$ is a tangent to the circle with equation $x^2 + y^2 - 4x - 8y - 20 = 0$ at the point P.

A second line with equation $y = kx - 4$ also passes through P.

Find the value of k , the gradient of this second line. **7**

[END OF SECTION B]

[END OF QUESTION PAPER]