DINGWALL ACADEMY

NATIONAL QUALIFICATIONS

Mathematics
Higher Mini-Prelim Examination 2011/2012

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** *or* **D**. *Indicate the correct answer by writing* **A**, **B**, **C** *or* **D** *opposite the number of the question on your answer paper.*
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. If k is a constant of integration then $\int \sqrt{4x+1}\ dx$ is

A $2(4x+1)^{-\frac{1}{2}} + k$

B $\frac{2}{3}(4x+1)^{\frac{3}{2}} + k$

C $\frac{1}{4}(4x+1)^{\frac{3}{2}} + k$

D $\frac{1}{6}(4x+1)^{\frac{3}{2}} + k$

2. If $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{b} = 8\boldsymbol{i} - 2\boldsymbol{j} - 6\boldsymbol{k}$. The value of $\boldsymbol{a}.\boldsymbol{b}$ is

A 4

B 0

C -4

D unknown without further information

3. The graph of $y = \log_2 4x$ crosses the x-axis at the point where x equals

A 2

B $0 \cdot 25$

C 4

D $0 \cdot 5$

4. Given that $x + 2$ is a factor of $x^3 - 2x^2 - 3x + c$, then the value of c is

A 10

B -10

C 6

D -6

5. Given that $|\boldsymbol{a}| = 2$, $|\boldsymbol{b}| = 3$ and $\boldsymbol{a}.\boldsymbol{b} = 4$, the value of $(2\boldsymbol{a} - \boldsymbol{b}).(\boldsymbol{a} + \boldsymbol{b})$ is

A 3

B -1

C 5

D 0

6. Given that $f(x) = \dfrac{1}{(2x-5)^3}$, then $f'(x)$ equals

A $\dfrac{-6}{(2x-5)^2}$

B $\dfrac{1}{6(2x-5)^4}$

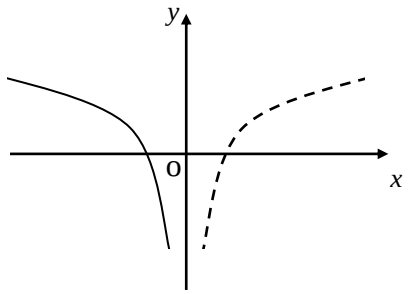
C $\dfrac{-6}{(2x-5)^4}$

D $\dfrac{-3}{(2x-5)^4}$

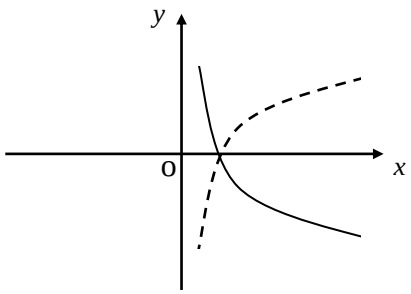
7. Part of the graph of $y = \log_{10} x$ is shown in each diagram below as a broken line.

Which diagram is most likely to show as an unbroken line part of the graph of $y = \log_{10} \frac{1}{x}$?

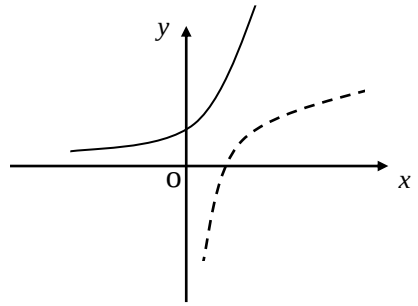
A



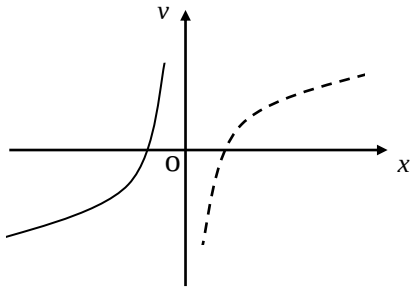
B



C



D



8. $\int_0^{\frac{\pi}{2}} \cos 2x \ dx$ is equal to

A 0

B -1

C $\frac{1}{2}$

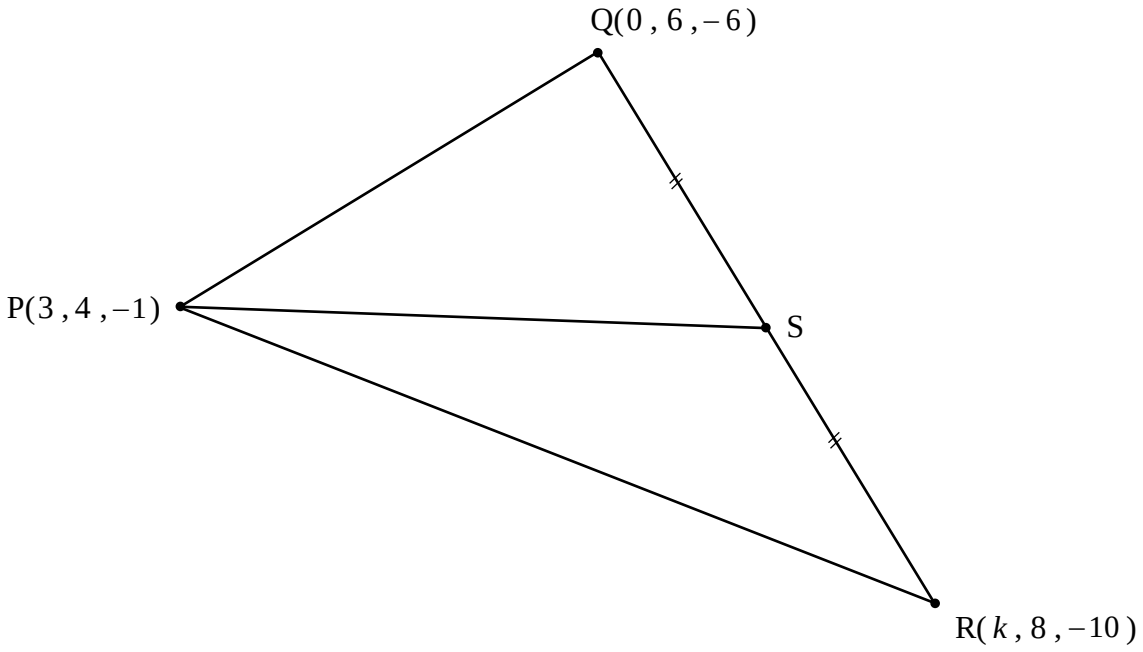
D 1

[END OF SECTION A]

SECTION B

ALL questions should be attempted

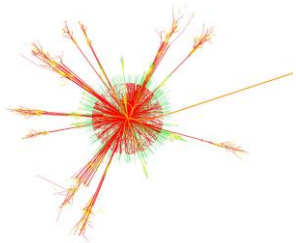
9. In the diagram P, Q, and R have coordinates P(3 , 4 , –1), Q(0 , 6 , – 6) and R(k , 8 , –10) respectively.



(a) Given that angle PQR is a right-angle, find the value of k. **4**

(b) Calculate the size of angle RPS where S is the mid-point of QR. **6**

10. A Baryon particle decays according to the formula $M_t = M_o\, e^{-0{\cdot}0009\, t}$, where M_o is the intitial mass of the substance and M_t is the mass remaining after t seconds.



Calculate , **to the nearest ten seconds**, how long a sample would take to lose 30% of its original mass. **5**

11. (a) Express $3\cos x^\circ + \sqrt{7}\sin x^\circ$ in the form $k\sin(x+a)^\circ$, where k and a are constants and $k > 0$. **5**

(b) Hence state the **minimum** value of f given that $f(x) = \dfrac{20}{3\cos x^\circ + \sqrt{7}\sin x^\circ}$. **1**

12. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n + 8$, where a is a constant.

(a) Given that $U_0 = 16$, show that, in terms of a, $U_2 = 8(2a^2 + a + 1)$. **2**

(b) Hence find a, where $a > 0$, given that $U_2 = 11$. **3**

13. A function is defined on a suitable domain as $h(x) = 2\sin 2x - \sqrt{3}\cos^2 x$.

Calculate the rate of change of this function at the point where $x = \frac{\pi}{3}$. **5**

14. Given that $\log_3(x+1) + 2\log_3 2 = 2$, find the value of x. **3**

[END OF SECTION B]

[END OF QUESTION PAPER]