NATIONAL
QUALIFICATIONS

Mathematics
Higher Mini-Prelim Examination 2010/2011

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} \mathrm{a}_1 \\ \mathrm{a}_2 \\ \mathrm{a}_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** *or* **D**. *Indicate the correct answer by writing* **A**, **B**, **C** *or* **D** *opposite the number of the question on your answer paper.*
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. If $f(x) = (2x-1)^4$ then $f'(1)$ equals

A 4

B 1

C 2

D 8

2. The **maximum** value of the function $g(x) = 3\sin x + 2\cos x$ is

A $\sqrt{13}$

B 5

C 0

D 2

3. The radius of the circle with equation $x^2 + y^2 + 4x - 2y = 4$ is

A 2

B 3

C 1

D $\sqrt{24}$

4. If k is a constant of integration then $\int \sin 4x \; dx$ is

A $-\cos 4x + k$

B $4\cos 4x + k$

C $-\frac{1}{4}\cos 4x + k$

D $\frac{1}{4}\cos 4x + k$

5. The value of $\log_{\sqrt{2}} 4$ is

A 2

B $4\sqrt{2}$

C $\frac{1}{4}$

D 4

6. Given that the vectors $\begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} p \\ -2 \\ 3 \end{pmatrix}$ are perpendicular, the value of p is

A 0

B 8

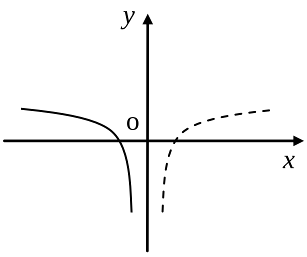
C 4

D -6

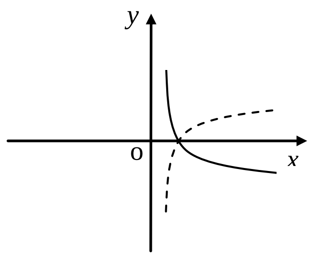
7. Part of the graph of $y = \log_{10} x$ is shown in each diagram below as a broken line.

Which diagram also shows, as a full line, part of the graph of $y = \log_{10} \frac{1}{x}$?

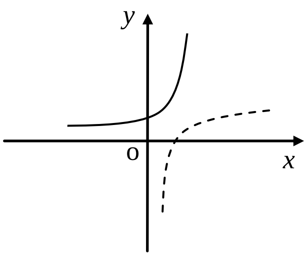
A



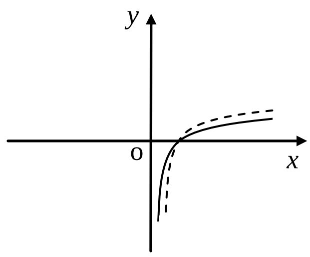
B



C



D



8. $\boldsymbol{a} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ g \end{pmatrix}$ is a **unit** vector. Which of the following could be the value of g?

A $\frac{1}{2}$

B 1

C -1

D $\frac{1}{\sqrt{2}}$

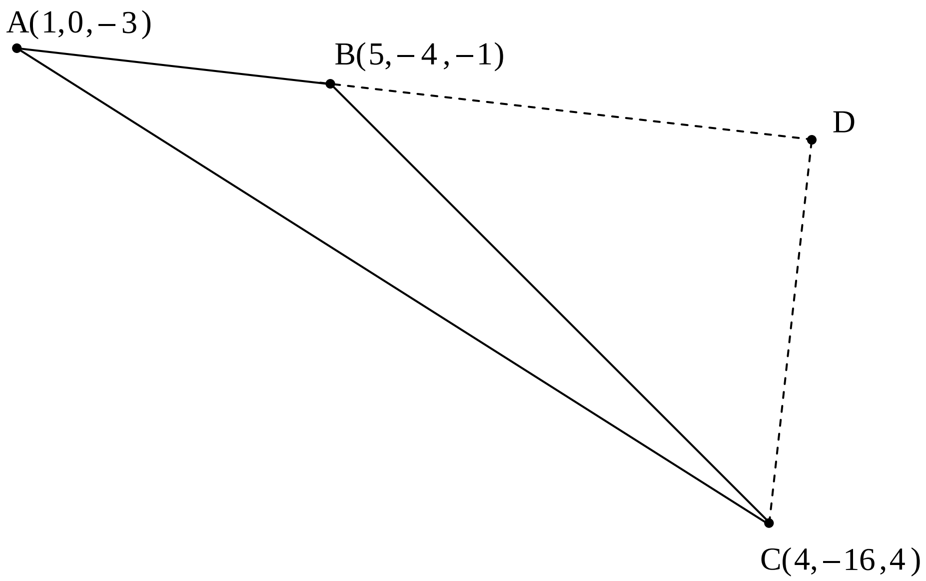
[**END OF SECTION A**]

SECTION B

ALL questions should be attempted

9. Triangle ABC has vertices $A(1, 0, -3)$, $B(5, -4, -1)$ and $C(4, -16, 4)$ respectively.

A, B and D are collinear such that $\frac{AB}{BD} = \frac{2}{3}$.



(a) Find the coordinates of D. **2**

(b) Hence show clearly that angle ADC is a right angle. **4**

(c) Prove that angle ABC is obtuse. **3**

10. A function is defined as $f(x) = 6\cos^2 \frac{1}{2}x^\circ + \sqrt{3}\sin x^\circ$.

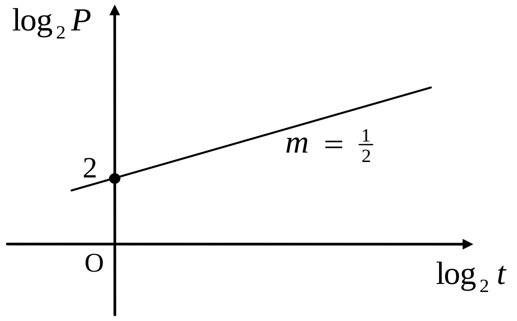
(a) By using the fact that $\cos^2 x^\circ = \frac{1}{2}(\cos 2x^\circ + 1)$ show clearly that this function can be expressed in the form

$$f(x) = 3\cos x^\circ + \sqrt{3}\sin x^\circ + 3.$$ **3**

(b) Express $3\cos x^\circ + \sqrt{3}\sin x^\circ + 3$ in the form $k\cos(x - \alpha)^\circ + 3$ where $0 < \alpha < 360$ and $k > 0$. **3**

(c) Hence solve the equation $f(x) = 0$ for $200 < x < 360$. **4**

11. The diagram, which is not drawn to scale, shows part of a graph of $\log_2 P$ against $\log_2 t$.
The straight line has a gradient of $\frac{1}{2}$ and passes through the point (0,2).



(a) Find an equation connecting t and P. **3**

(b) Hence show clearly that when $P = \sqrt{8} + 4$, t takes the value $\frac{1}{2}(3 + 2\sqrt{2})$ **3**

12. Given that $(x+1)$ and $(x-3)$ are both factors of $2x^3 - 5x^2 + ax + b$, find a and b. **4**

13. (a) Given that $y = \sqrt{3}(\sin^2 x - \cos 2x)$, show clearly that

$$\frac{dy}{dx} = \sqrt{3}(3\sin 2x).$$ **3**

(b) Hence find the gradient of the tangent to the curve $y = \sqrt{3}(\sin^2 x - \cos 2x)$ at the point where $x = \frac{\pi}{6}$. **2**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]