DINGWALL ACADEMY

NATIONAL QUALIFICATIONS

Mathematics
Higher Mini-Prelim Examination 2009/2010

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** *or* **D**. *Indicate the correct answer by writing* **A**, **B**, **C** *or* **D** *opposite the number of the question on your answer paper.*
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. The function $f(x) = 2\sin x^\circ + \cos x^\circ$ has a **minimum** value of

A -2

B 0

C -3

D $-\sqrt{5}$

2. Which of the following is a correct assumption from the statement $\log_b a = c$?

A $a^c = b$

B $c^a = b$

C $b^c = a$

D $c^b = a$

3. What is the value of $\int_0^{\pi} \sin x \; dx$?

A -2

B $+1$

C $+2$

D 0

4. P and Q have position vectors $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

The length of PQ is

A 1

B $\sqrt{17}$

C $\sqrt{21}$

D $\sqrt{13}$

5. Given that $\cos P = \frac{1}{\sqrt{6}}$, where $0 < p < \frac{\pi}{2}$, the value of $\cos 2P$ is

A $\frac{1}{\sqrt{3}}$

B $\frac{2}{\sqrt{6}}$

C $\frac{\sqrt{5}}{\sqrt{6}}$

D $-\frac{2}{3}$

6. An equation is such that $\log x + \log(x+1) = \log 6$, where $x > 0$.

The value of *x* is

A 2

B 1

C 3

D 6

7. The gradient of the tangent to the curve $y = \sin x^\circ$ at the point where $x = \frac{\pi}{3}$ radians is

A $\frac{\sqrt{3}}{2}$

B $\frac{1}{2}$

C $-\frac{1}{2}$

D 0

8. Vectors ***a*** and ***b*** are such that $|\boldsymbol{a}| = |\boldsymbol{b}| = 2$ with *P* being the angle between the vectors.

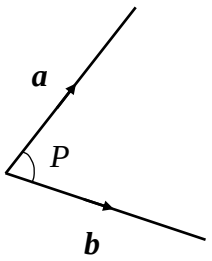
If $\boldsymbol{a} \cdot \boldsymbol{b} = 0 \cdot 8$, the value of $\cos P$ is

A $3 \cdot 2$

B $0 \cdot 4$

C $0 \cdot 2$

D $0 \cdot 05$



[END OF SECTION A]

SECTION B

ALL questions should be attempted

9. A function is defined on a suitable domain as $f(x) = \dfrac{-16}{(2x-1)^2}$.

(a) Show clearly that the derivative of this function can be written in the form

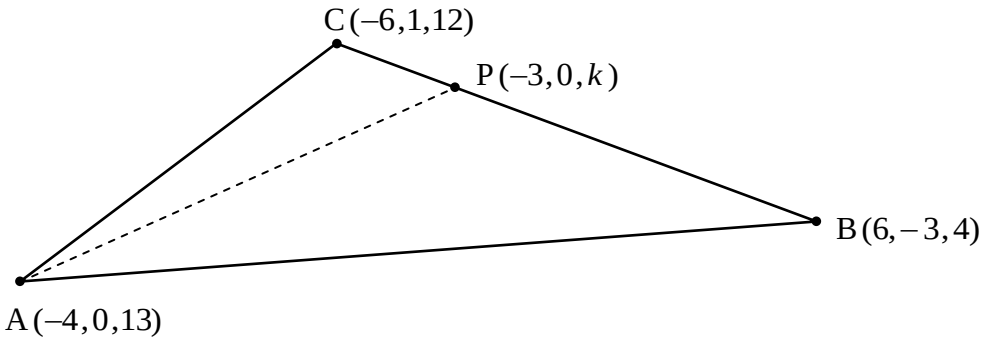
$$f'(x) = \frac{k}{(2x-1)^n}$$

and write down the values of *k* and *n*. **4**

(b) Hence find *x* when $f'(x) = 1$ and $x > 0$. **3**

10. In the diagram below A, B and C have coordinates $(-4, 0, 13)$, $(6, -3, 4)$ and $(-6, 1, 12)$ respectively.

P lies on BC and has coordinates $(-3, 0, k)$



(a) Find the value of *k*. **3**

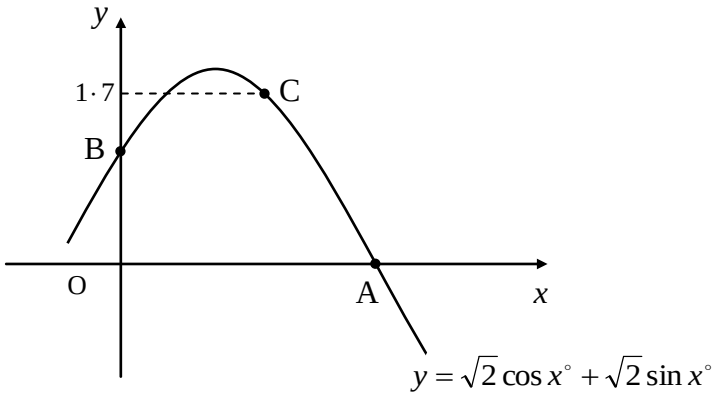
(b) Hence calculate the size of angle APB. **5**

11. A formulae for mass decay is given as $M_t = M_0 e^{-0{\cdot}02t}$, where *t* is time elapsed in hours, M_0 is the initial mass in grams and M_t is the mass remaining after *t* hours.

How long will it take for an initial mass of 40 grams to decay down to 28 grams?
Give your answer correct to the nearest minute. **5**

12. If $\frac{dy}{dx} = \sqrt{4x+1}$, find an expression for y in terms of x given that $y = 9 \cdot 5$ when $x = 2$. **5**

13. Part of the graph of $y = \sqrt{2}\cos x^\circ + \sqrt{2}\sin x^\circ$ is shown below.



(a) Express $y = \sqrt{2}\cos x^\circ + \sqrt{2}\sin x^\circ$ in the form $y = k\cos(x-a)^\circ$, where $k > 0$. **3**

(b) Hence state the coordinates of A and B rounding the coordinates to **3 significant figures** where necessary. **2**

(c) By solving the equation $\sqrt{2}\cos x^\circ + \sqrt{2}\sin x^\circ = 1 \cdot 7$, find the coordinates of point C. **4**

[END OF SECTION B]

[END OF QUESTION PAPER]