# DINGWALL ACADEMY

**Mathematics**
**Higher Mini-Prelim Examination 2008/2009**

NATIONAL QUALIFICATIONS

**Assessing Unit 3 + revision from Units 1 & 2**

**Time allowed - 1 hour 10 minutes**

**Read carefully**

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

**FORMULAE LIST**

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Trigonometric formulae:**

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

**Scalar Product:** $\boldsymbol{a} . \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where $\theta$ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} . \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

**Table of standard derivatives:**

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a \cos ax$ |
| $\cos ax$ | $-a \sin ax$ |

**Table of standard integrals:**

| $f(x)$ | $\int f(x)\, dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a} \cos ax + C$ |
| $\cos ax$ | $\frac{1}{a} \sin ax + C$ |

## SECTION A

*In this section the correct answer to each question is given by one of the alternatives* **A**, **B**, **C** *or* **D**. *Indicate the correct answer by writing* **A**, **B**, **C** *or* **D** *opposite the number of the question on your answer paper.*
*Rough working may be done on the paper provided. 2 marks will be given for each correct answer.*

**1.** A is the point $(-4, 6, 5)$ and B is the point $(-1, 3, 2)$. The components of $\overrightarrow{AB}$ are

**A** $\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ **B** $\begin{pmatrix} -5 \\ 9 \\ 7 \end{pmatrix}$

**C** $\begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$ **D** $\begin{pmatrix} 5 \\ -9 \\ -7 \end{pmatrix}$

**2.** The gradient of the tangent to the curve $y = 3\sin 2x$ at the point where $x = \frac{\pi}{6}$ is

**A** $3\sqrt{3}$

**B** $3$

**C** $-3$

**D** $-3\sqrt{3}$

**3.** The circle $x^2 + y^2 + 11x + 7y + 10 = 0$ cuts the $x$-axis at the points P and Q.

The length of PQ is

**A** 3

**B** 7

**C** 9

**D** 11

**4.** Given that $C$ is a constant of integration, then $\int (4x+3)^{-\frac{1}{2}}\, dx$ equals

**A** $(4x+3)^{\frac{1}{2}} + C$

**B** $\frac{1}{2}(4x+3)^{\frac{1}{2}} + C$

**C** $\frac{1}{4}(4x+3)^{\frac{1}{2}} + C$

**D** $-2(4x+3)^{-\frac{3}{2}} + C$

**5.** The derivative of $(3-4x)^3$ with respect to $x$ is

**A** $-\dfrac{(3-4x)^4}{16}$

**B** $\dfrac{(3-4x)^4}{4}$

**C** $-(3-4x)^4$

**D** $-12(3-4x)^2$

**6.** Vector $\boldsymbol{a}$ has components $\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \\ k \end{pmatrix}$.

If $|\boldsymbol{a}| = 4$, then the value of $k$ is

**A** 3

**B** $-1$

**C** $-13$

**D** $\sqrt{3}$

**7.** Solve $\log_3 3x + \log_3 x = 3$, for $x$ where $x > 0$.

**A** 1

**B** $\frac{27}{4}$

**C** 3

**D** $\frac{3}{4}$

**8.** The maximum value of $3\sin x - 4\cos x + 5$ is

**A** 10

**B** 0

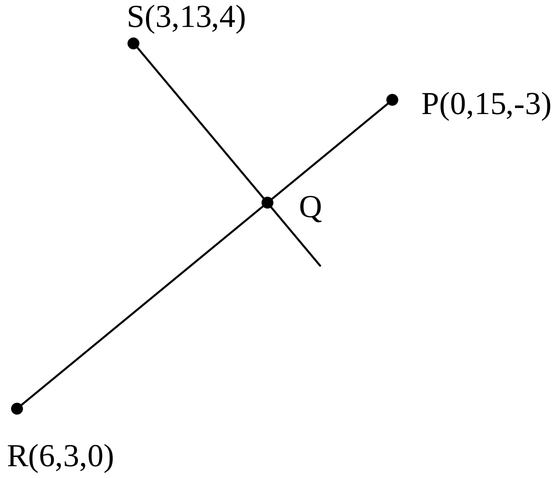
**C** 4

**D** $-5$

[ **END OF SECTION A** ]

## SECTION B
**ALL questions should be attempted**

**9.** Consider the diagram below.




(a) Given that Q divides PR in the ratio $1:2$, find the coordinates of Q. **3**

(b) Hence prove that angle SQR is a right angle. **4**

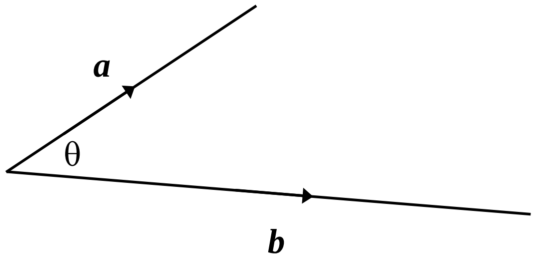
**10.** Evaluate $\int_0^1 \frac{6}{(3-2x)^2}\ dx$. **5**

**11.** Solve the equation $\sin x^\circ + 3\cos x^\circ = 2$ for $0 < x \leq 360$. **6**

**12.** Find the coordinates of the point on the curve $y = x^3 - x^2 - 4x + 2$ where the gradient of the tangent is 1 and $x < 0$. **4**

**13.** The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ where $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$.

The angle between the vectors is $\theta$.




(a) Show clearly that $\cos\theta = \frac{4}{5}$. **3**

(b) Hence, or otherwise, find the exact value of $\cos 2\theta$. **2**

**14.** The mass of radium-226 remaining after a decay period of $t$ years can be calculated using the formula


$M_t = M_0 e^{kt}$, where $M_0$ is the initial mass, $M_t$ is the mass remaining after $t$ years and $k$ is a constant.

(a) Find the value of the constant $k$, given that a sample of radium-226 takes 500 years to **decay to 80%** of its initial mass.
**Give your answer correct to 2 significant figures.** **5**

(b) Hence calculate the approximate percentage mass remaining, of a sample of radium-226, after a period of 5 thousand years.
**Give your answer correct to the nearest percent.** **2**

[ **END OF SECTION B** ]

[ **END OF QUESTION PAPER** ]