

Dingwall Academy

Mathematics
Higher Mini-Prelim Examination 2007/2008

**NATIONAL
QUALIFICATIONS**

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$ $\cos ax$	$a \cos ax$ $-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$ $\cos ax$	$-\frac{1}{a} \cos ax + C$ $\frac{1}{a} \sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** or **D**. Indicate the correct answer by writing **A**, **B**, **C** or **D** opposite the number of the question on your answer paper.

Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. A vector is defined as

$$\mathbf{v} = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5} \end{pmatrix}.$$

The magnitude of this vector is

- A** 3
B $2 + \sqrt{5}$
C 9
D unknown

2. $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$ is equal to

- A** $\frac{1}{2}$
B -1
C 0
D 2

3. The maximum value of $2\cos x - 3\sin x$ is

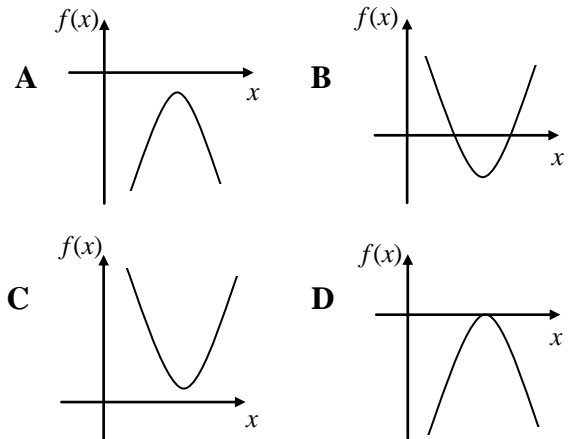
- A** 5
B -1
C $\sqrt{5}$
D $\sqrt{13}$

4. The exact value of $\log_9 27$ is

- A** $\frac{1}{3}$
B $\frac{2}{3}$
C $\frac{3}{2}$
D 3

5. A quadratic function f , where $f(x) = ax^2 + bx + c$, is such that $a < 0$ and $b^2 - 4ac < 0$.

Which of the following could be a possible sketch of the graph of this function?



6. What value of x makes the vectors

$$\begin{pmatrix} -2 \\ 4 \\ 10 \end{pmatrix} \text{ and } \begin{pmatrix} -3 \\ 6 \\ x \end{pmatrix} \text{ perpendicular to each other?}$$

- A** 15
B -3
C 9
D no possible value

7. If $f(x) = \sin^3 x$ then $f'(x)$ equals

- A** $3\sin^2 x$
B $3\cos^2 x$
C $3\sin^2 x \cos x$
D $-3\sin^2 x \cos x$

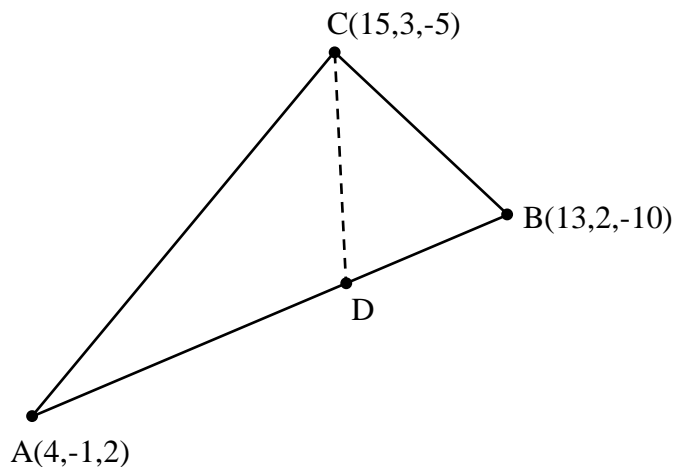
8. The graph of $y = \log_2 x$ cuts the x -axis at

- A** (0,0)
B (1,0)
C (2,0)
D (0,1)

SECTION B

ALL questions should be attempted

9. Triangle ABC has vertices A(4,-1,2), B(13,2,-10) and C(15,3,-5) as shown.
Point D lies on side AB.



- (a) Given that D divides the line AB in the ratio 2:1, find the coordinates of D. **3**
- (b) Hence calculate the size of angle CDA. **5**
10. Given $f(x) = \frac{9}{1-4x}$ where $x \neq \frac{1}{4}$, find the value of $f'(1)$. **4**

11. The noise level, N decibels, emitting from a siren as it slowly gains and loses volume is given by the formula

$$N = 2 \cos t^\circ + 4\sqrt{2} \sin t^\circ + 30,$$

where t is the time elapsed, in seconds, from switch on.



- (a) Express N in the form $N = k \sin(t + \alpha)^\circ + 30$, where $k > 0$ and $0 \leq \alpha \leq 90$. **4**
- (b) Hence calculate how many seconds the siren takes to first reach a noise level of 34 decibels.
Give your answer correct to 3 significant figures. **3**

12. (a) Given that $3\log_x y = \log_x y^2 + 2$, find a relationship connecting x and y . 4
- (b) Hence find the two values of y when $x = y - 2$. 3

13. (a) A **linear** function, f , is such that $f(-1) = -3$ and $f(4) = 7$.
Find a formula for this function in terms of x . 3

- (b) Given that a second function, g , has as its formula $g(x) = x^3$, evaluate

$$\int_1^2 [g(f(x))] dx \quad \text{5}$$

[END OF SECTION B]

[END OF QUESTION PAPER]