



**2012 Mathematics**

**Higher**

**Finalised Marking Instructions**

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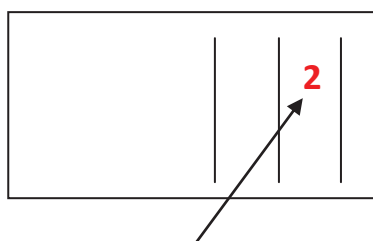
## General Comments

These marking instructions are for use with the 2012 Higher Mathematics Examination.

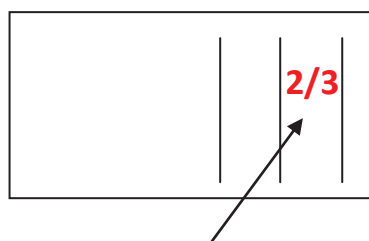
For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which are commonly seen throughout the marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general markers should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

- 1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.
- 2 Award one mark for each •. There are no half marks.
- 3 The mark awarded for **each part** of a question should be entered in the **outer** right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, **as a whole number**, should be written.



Marks in this column -  
whole numbers only



Do not record marks on  
scripts in this manner.

- 4 Where a candidate has not been awarded any marks for a question, or part of a question, 0 should be written in the right hand margin against their answer. It should not be left blank.
- 5 Every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol should have a tick placed in the bottom right hand margin.
- 6 Where the solution to part of a question is fragmented and continues later in the script, the marks should be recorded at the end of the solution. This should be indicated with a down arrow ( $\downarrow$ ), in the margin, at the earlier stages.
- 7 Working subsequent to an error must be **followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.
- 8 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

## 9 Marking Symbols

No comments or words should be written on scripts. Please use the following and the symbols indicated on the welcome letter and from comment 6 on the previous page.

✓ A tick should be used where a piece of working is correct and gains a mark. Markers must check through the whole of a response, ticking the work only where a mark is awarded.

— X At the point where an error occurs, the error should be underlined and a cross used to indicate where a mark has not been awarded. If no mark is lost the error should only be underlined, i.e. a cross is only used where a mark is not awarded.

✗ A cross-tick should be used to indicate “correct” working where a mark is awarded as a result of **follow through** from an error.

✗ A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been **eased**.

~~~~ A tilde should be used to indicate a minor error which is not being penalised, e.g. **bad form**.

✓~~~~ This should be used where a candidate is given the **benefit of the doubt**.

^ A roof should be used to show that something is missing, such as part of a solution or a crucial step in the working.

These will help markers to maintain consistency in their marking and will assist the examiners in the later stages of SQA procedures.

The examples below illustrate the use of the marking symbols .

### Example 1

$$y = x^3 - 6x^2$$

$$\frac{dy}{dx} = 3x^2 - 12 \quad \text{X}$$

$$3x^2 - 12 = 0 \quad \text{X}$$

$$x = 2 \quad \wedge$$

$$y = -16 \quad \text{X}$$

•<sup>1</sup> ✓

•<sup>2</sup> X

•<sup>3</sup> ✗

•<sup>4</sup> ^

•<sup>5</sup> ✗

### Example 2

$$A(4,4,0), B(2,2,6), C(2,2,0)$$

$$\overrightarrow{AB} = \underline{\mathbf{b+a}} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \quad \text{X} \quad \bullet^1$$

$$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} \quad \text{X} \quad \bullet^2 \text{ (repeated error)}$$

### Example 3

$$3 \sin x - 5 \cos x$$

$$k \sin x \cos a - \cos x \sin a \quad \text{✓} \bullet^1$$

$$k \cos a = 3, k \sin a = 5 \quad \text{✓} \bullet^2$$

### Example 4

$$4 \left| \begin{array}{cccc} 1 & -5 & 2 & 8 \quad \text{✓} \bullet^1 \\ & 4 & -4 & -8 \\ & 1 & 1 & -2 \quad \text{✓} \bullet^2 \\ & & & 0 \quad \text{✓} \bullet^2 \end{array} \right.$$

Since the remainder is 0,  $x - 4$  must be a factor. ✓ •<sup>3</sup>

$$(x^2 - x - 2) \quad \text{✓} \bullet^4$$

$$(x - 4)(x + 1)(x - 2) \quad \text{✓} \bullet^5$$

$$x = 4 \text{ or } x = -1 \text{ or } x = 2 \quad \text{✓} \bullet^6$$

- 10** In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g.  $6 \times 6 = 12$ , candidates lose the opportunity of gaining a mark. But note example 4 in comment 9 and comment 11.
- 11** Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, e.g.

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation.

$$\begin{array}{l}
 x^2 + 5x + 7 = 9x + 4 \quad \checkmark \\
 \underline{x - 4x + 3 = 0} \quad \times \\
 x = 1 \quad \times
 \end{array}$$

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$$\begin{array}{l}
 x^2 + 5x + 7 = 9x + 4 \quad \checkmark \\
 x - 4x + 3 = 0 \quad \checkmark \\
 (x-3)(x-1) = 0 \\
 x = 1 \text{ or } 3 \quad \checkmark
 \end{array}$$

**12 Cross marking**

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme:  $\bullet^5 x = 2, x = -4$   
 $\bullet^6 y = 5, y = -7$

Cross marked:  $\bullet^5 x = 2, y = 5$   
 $\bullet^6 x = -4, y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

**13** In final answers, numerical values should be simplified as far as possible.

Examples:  $\frac{15}{12}$  must be simplified to  $\frac{5}{4}$  or  $1\frac{1}{4}$        $\frac{43}{1}$  must be simplified to 43  
 $\frac{15}{0.3}$  must be simplified to 50       $\frac{4/5}{3}$  must be simplified to  $\frac{4}{15}$   
 $\sqrt{64}$  must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

**14** Regularly occurring responses (ROR) are shown in the marking instructions to help mark common and/or non-routine solutions. RORs may also be used as a guide in marking similar non-routine candidate responses.

**15** Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form;
- Repeated error within a question, but not between questions or papers.

- 16** In any ‘Show that . . .’ question, where the candidate has to arrive at a formula, the last mark of that part is not available as a follow through from a previous error.
- 17** All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate’s response. Marks may still be available later in the question so reference must be made continually to the marking instructions. All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.
- 18** In the **exceptional** circumstance where you are in doubt whether a mark should or should not be awarded, consult your Team Leader (TL).
- 19** Scored out working which **has not been replaced** should be marked where still legible. However, if the scored out working **has been replaced**, only the work which has not been scored out should be marked.
- 20** A valid approach, within Mathematical problem solving, is to try different strategies. Where this occurs, all working should be marked. The mark awarded to the candidate is from the **highest** scoring strategy. This is distinctly different from the candidate who gives two or more solutions to a question/part of a question, deliberately leaving all solutions, hoping to gain some benefit. All such contradictory responses should be marked and the **lowest** mark given.
- 21** It is of great importance that the utmost care should be exercised in totalling the marks.  
The recommended procedure is as follows:  
Step 1     Manually calculate the total from the candidate’s script.  
Step 2     Check this total using the grid issued with these marking instructions.  
Step 3     In EMC, enter the marks and obtain a total, which should now be compared to the manual total.  
This procedure enables markers to identify and rectify any errors in data entry before submitting each candidate’s marks.
- 22** **The candidate’s script for Paper 2 should be placed inside the script for Paper 1, and the candidate’s total score (i.e. Paper 1 Section B + Paper 2) written in the space provided on the front cover of the script for Paper 1.**
- 23** In cases of difficulty, covered neither in detail nor in principle in these instructions, markers should contact their TL in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with the TL. Further details of PA Referrals can be found in The General Marking Instructions.

1 Functions  $f$  and  $g$  are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$

(a) Find expressions for:

- (i)  $f(g(x))$ ;
- (ii)  $g(f(x))$ .

3

**Generic Scheme**

**Illustrative Scheme**

1 (a)

- |                                                                                                                                                                                                               |                                                                                                                                                                                                                                          |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> <li>•<sup>1</sup> ic start composite process</li> <li>•<sup>2</sup> ic correct substitution into expression</li> <li>•<sup>3</sup> ic complete second composite</li> </ul> | <ul style="list-style-type: none"> <li>•<sup>1</sup> e.g. <math>f(x+4)</math>      <b>stated, or implied by</b> •<sup>2</sup></li> <li>•<sup>2</sup> <math>(x+4)^2 + 3</math></li> <li>•<sup>3</sup> <math>x^2 + 3 + 4</math></li> </ul> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

3

**Notes**

- Candidates must clearly identify which of their answers are  $f(g(x))$  and  $g(f(x))$ ; the minimum evidence for this could be as little as using (i) and (ii) as labels.
- Candidates who interpret the composite functions as either  $f(x) \times g(x)$  or  $f(x) + g(x)$ , do not gain any marks.

**Regularly occurring responses**

**Response 1 :** The first two marks are for **either**  $f(g(x))$  **or**  $g(f(x))$  correct. The third mark is for the other composite function.

**Candidate A**

$$f(g(x)) = (x+4)^2 + 3 \quad \checkmark \bullet^1 \checkmark \bullet^2$$

$$g(f(x)) = x^2 + 12 \quad \times \bullet^3$$

2 marks out of 3

**Candidate B**

$$f(g(x)) = (x+7)^2 \quad \times \bullet^3$$

$$g(f(x)) = x^2 + 7 \quad \checkmark \bullet^1 \checkmark \bullet^2$$

2 marks out of 3

**Response 2 :** Interpreting  $f(g(x))$  as  $g(f(x))$  and vice versa. A maximum of 2 marks are available.

**Candidate C**

$$f(g(x)) = x^2 + 7 \quad \times \bullet^1 \times \bullet^2$$

$$g(f(x)) = (x+4)^2 + 3 \quad \times \bullet^3$$

2 marks out of 3

**Candidate D**

$$f(g(x)) = x^2 + 7 \quad \times \bullet^1 \times \bullet^2$$

1 mark out of 3

**Response 3 :** Identifying  $f(g(x))$  and  $g(f(x))$

**Candidate E**

$$(x+4)^2 + 3 \quad \times \bullet^1 \checkmark \bullet^2$$

$$x^2 + 7 \quad \checkmark \bullet^3$$

2 marks out of 3

**Candidate F**

$$x^2 + 7 \quad \times \bullet^1 \times \bullet^2$$

$$(x+4)^2 + 3 \quad \times \bullet^3$$

1 mark out of 3

**Candidate G**

$$x^2 + 7 \quad \text{ONLY}$$

$$\text{or } (x+4)^2 + 3 \quad \text{ONLY}$$

0 marks out of 3

**Candidate H**

$$(i) (x+4)^2 + 3 \quad \checkmark \bullet^1 \checkmark \bullet^2$$

$$(ii) x^2 + 7 \quad \checkmark \bullet^3$$

3 marks out of 3

## Generic Scheme

## Illustrative Scheme

1 (b)

Method 1 : Discriminant

- <sup>4</sup> pd obtain a quadratic expression
- <sup>5</sup> ss know to and use discriminant
- <sup>6</sup> ic interpret result

Method 2 : Quadratic Formula

- <sup>4</sup> pd obtain a quadratic expression
- <sup>5</sup> ss know to and use quadratic formula
- <sup>6</sup> ic interpret result

Method 1 : Discriminant

- <sup>4</sup>  $2x^2 + 8x + 26$
- <sup>5</sup>  $8^2 - 4 \times 2 \times 26$  or  $4^2 - 4 \times 1 \times 13$  **stated, or implied by** •<sup>6</sup>
- <sup>6</sup>  $-144 < 0$  or  $-36 < 0$  so no real roots

Method 2 : Quadratic Formula

- <sup>4</sup>  $2x^2 + 8x + 26$
- <sup>5</sup>  $\frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 26}}{2 \times 2}$  **stated, or implied by** •<sup>6</sup>
- <sup>6</sup>  $\sqrt{-144}$  not possible so no real roots

3

## Notes

3. Candidates who use  $f(x) \times g(x)$  can gain no marks in (b) as a cubic will be obtained.
4. Candidates who use  $f(x) + g(x)$  do not gain •<sup>4</sup> (eased) but •<sup>5</sup> and •<sup>6</sup> are available as follow through marks.
5. In method 1, any other formula masquerading as a discriminant cannot gain •<sup>5</sup> and •<sup>6</sup>.
6. •<sup>4</sup>, •<sup>5</sup> and •<sup>6</sup> are only available if  $f(g(x)) + g(f(x))$  simplifies to a quadratic expression of the form  $ax^2 + bx + c$ , with  $b$  and  $c$  both non-zero.
7. •<sup>6</sup> is only available for a numerical value, calculated correctly from the candidate's response at •<sup>4</sup>, and leading to no real roots.
8. Do not accept for •<sup>6</sup>:
  - 'no roots' in lieu of 'no real roots'
  - 'maths error' or 'ma error'.
9. Candidates who use the word derivative instead of discriminant should not be penalised.

## Regularly occurring responses

**Response 4:** Candidates who do not simplify the value of their discriminant**Candidate I**

$$8^2 - 4 \times 2 \times 26 \quad \checkmark \quad \bullet^5 \quad \checkmark$$

$$= 64 - 208 < 0 \text{ so no real roots} \quad \bullet^6 \quad \times$$

**Response 5:** Acceptable communication marks**Method 1****Candidate J**

$$\sqrt{8^2 - 4 \times 2 \times 26} \quad \checkmark \quad \bullet^5$$

$$= \sqrt{-144}$$

not valid  
so no real roots  $\checkmark \bullet^6$

**Candidate L**

no real roots if  $b^2 - 4ac < 0$   
 $64 - 208 = -144 \quad \checkmark \bullet^6$   
 $\checkmark \bullet^5$

**Candidate K**

$$\text{Discriminant} = \sqrt{8^2 - 4 \times 2 \times 26} \quad \checkmark \bullet^5$$

$$= \sqrt{-144}$$

can't find root of negative  
so no real roots  $\checkmark \bullet^6$

**Method 2****Candidate M**

$$\frac{-(-4) \pm \sqrt{8^2 - 4 \times 2 \times 26}}{2 \times 2} \quad \checkmark \bullet^5$$

$$= \frac{4 \pm \sqrt{-144}}{4}$$

no  $\sqrt{-ve}$   
so no real roots  $\checkmark \bullet^6$

- 2 (a) Relative to a suitable set of coordinate axes, diagram 1 shows the line  $2x - y + 5 = 0$  intersecting the circle  $x^2 + y^2 - 6x - 2y - 30 = 0$  at the points P and Q.



Find the coordinates of P and Q.

6

Diagram 1

### Generic Scheme

### Illustrative Scheme

2 (a)

- <sup>1</sup> ss rearrange linear equation
- <sup>2</sup> ss substitute into circle
- <sup>3</sup> pd express in standard form
- <sup>4</sup> pd start to solve
- <sup>5</sup> ic state roots
- <sup>6</sup> pd determine corresponding  $y$ -coordinates

Substituting for  $y$

- <sup>1</sup>  $y = 2x + 5$  **stated, or implied by** •<sup>2</sup>
- <sup>2</sup>  $\dots (2x + 5)^2 \dots - 2(2x + 5) \dots$
- <sup>3</sup>  $5x^2 + 10x - 15$  } = 0 must appear at the •<sup>3</sup>
- <sup>4</sup> e.g.  $5(x + 3)(x - 1)$  } or •<sup>4</sup> stage to gain •<sup>3</sup>.
- <sup>5</sup>  $x = -3$  and  $x = 1$
- <sup>6</sup>  $y = -1$  and  $y = 7$

Substituting for  $x$

- <sup>1</sup>  $x = \frac{y - 5}{2}$  **stated, or implied by** •<sup>2</sup>
- <sup>2</sup>  $\left(\frac{y - 5}{2}\right)^2 \dots - 6\left(\frac{y - 5}{2}\right) \dots$
- <sup>3</sup>  $5y^2 - 30y - 35$  } = 0 must appear at the •<sup>3</sup>
- <sup>4</sup> e.g.  $5(y + 1)(y - 7)$  } or •<sup>4</sup> stage to gain •<sup>3</sup>.
- <sup>5</sup>  $y = -1$  and  $y = 7$
- <sup>6</sup>  $x = -3$  and  $x = 1$

6

### Notes

- At •<sup>4</sup> the quadratic must lead to two real distinct roots for •<sup>5</sup> and •<sup>6</sup> to be available.
- Cross marking is available here for •<sup>5</sup> and •<sup>6</sup>.
- Candidates do not need to distinguish between points P and Q.

### Regularly occurring responses

Response 1 : Solving quadratic equation

Candidate A

$\checkmark$  •<sup>1</sup>  $\checkmark$  •<sup>2</sup>  
 $5x^2 + 10x + 5 = 0$   $\times$  •<sup>3</sup>  
 $5(x + 1)(x + 1)$   $\checkmark$  •<sup>4</sup>  
 $x = -1$   $\times$  •<sup>5</sup>  
 $y = 3$   $\times$  •<sup>6</sup>

Candidate B

$y = 2x + 5$   $\checkmark$  •<sup>1</sup>  
 $x^2 + (2x + 5)^2 - 6x - 2(7x + 5) - 30 = 0$   $\times$  •<sup>2</sup>  
 $5x^2 - 15 = 0$   $\checkmark$  •<sup>3</sup>  
 $x^2 = 3$   $\checkmark$  •<sup>4</sup>  
 $x = \pm\sqrt{3}$   $\times$  •<sup>5</sup>  
 $y = 8.5, 1.5$   $\checkmark$  •<sup>6</sup>

Candidate C

$\checkmark$  •<sup>1</sup>  $\checkmark$  •<sup>2</sup>  
 $5x^2 + 10x - 15 = 0$   $\checkmark$  •<sup>3</sup>  
 $5x^2 + 10x = 15$   
 $5x(x + 2) = 15$   $\times$  •<sup>4</sup>  
 $x(x + 2) = 3$   
 $x = 3$   $x = 1$   $\times$  •<sup>5</sup>  
 $y = 11$   $y = 7$   $\times$  •<sup>6</sup>

Cross marking is **not** available here for •<sup>5</sup> and •<sup>6</sup>, as there are no distinct roots. See Note 1.



- 2 (b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.  
Determine the equation of this second circle.

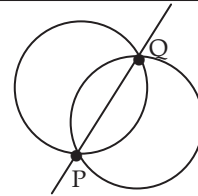


Diagram 2

6

**Generic Scheme**

**Illustrative Scheme**

2 (b)

- <sup>7</sup> ic centre of original circle
  - <sup>8</sup> pd radius of original circle
- Method 1 : Using midpoint
- <sup>9</sup> ss midpoint of chord
  - <sup>10</sup> ss evidence for finding new centre
  - <sup>11</sup> ic centre of new circle
  - <sup>12</sup> ic equation of new circle
- Method 2 : Stepping out using P and Q
- <sup>9</sup> ss evidence of  $C_1$  to P or  $C_1$  to Q
  - <sup>10</sup> ss evidence of Q to  $C_2$  or P to  $C_2$
  - <sup>11</sup> ic centre of new circle
  - <sup>12</sup> ic equation of new circle

- <sup>7</sup> (3, 1)
  - <sup>8</sup>  $\sqrt{40}$  Accept  $r^2 = 40$
- Method 1 : Using midpoint
- <sup>9</sup> (-1, 3)
  - <sup>10</sup> e.g. stepping out or midpoint formula
  - <sup>11</sup> (-5, 5)
  - <sup>12</sup>  $(x + 5)^2 + (y - 5)^2 = 40$
- Method 2 : Stepping out using P and Q
- <sup>9</sup> e.g. stepping out or vector approach
  - <sup>10</sup> e.g. stepping out or vector approach
  - <sup>11</sup> (-5, 5)
  - <sup>12</sup>  $(x + 5)^2 + (y - 5)^2 = 40$

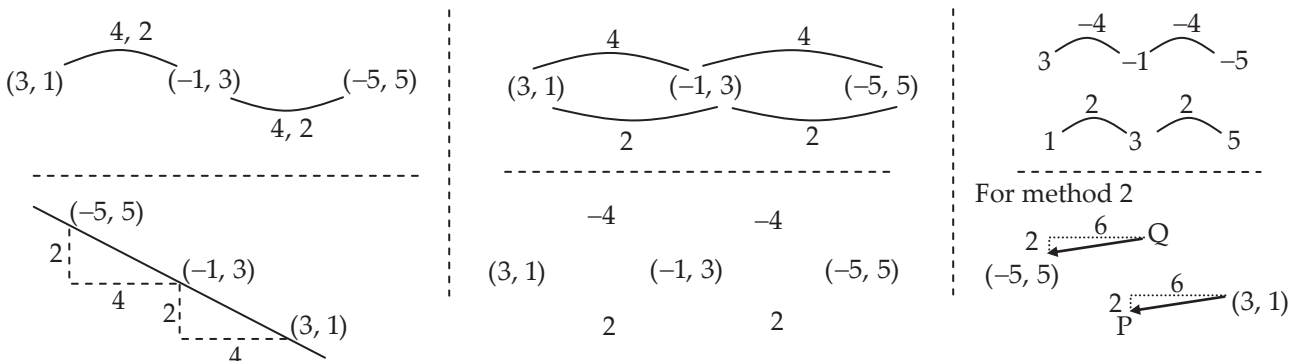
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**Notes**

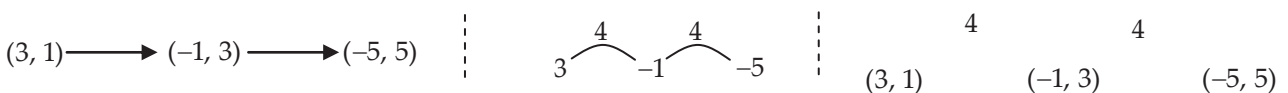
4. The evidence for •<sup>7</sup> and •<sup>8</sup> may appear in (a).
5. Centre (-5, 5) **without working** in method 1 may still gain •<sup>12</sup> but not •<sup>10</sup> or •<sup>11</sup>, in method 2 may still gain •<sup>12</sup> but not •<sup>9</sup>, •<sup>10</sup> or •<sup>11</sup>.  
Any other centre **without working** in method 1 does not gain •<sup>10</sup>, •<sup>11</sup> or •<sup>12</sup>, in method 2 does not gain •<sup>9</sup>, •<sup>10</sup>, •<sup>11</sup> or •<sup>12</sup>.
6. The centre must have been clearly indicated before it is used at the •<sup>12</sup> stage.
7. Do not accept e.g.  $\sqrt{40^2}$  or  $39.69$ , or any other decimal approximations for •<sup>12</sup>.
8. The evidence for •<sup>8</sup> may not appear until the candidate states the radius or equation of the second circle.

**Regularly occurring responses**

**Response 2 :** Examples of evidence for stepping out for •<sup>10</sup> in method 1 or •<sup>9</sup> or •<sup>10</sup> in method 2



**Response 3 :** Examples of evidence which do not gain •<sup>10</sup> in method 1 for stepping out



3 A function  $f$  is defined on the domain  $0 \leq x \leq 3$  by  $f(x) = x^3 - 2x^2 - 4x + 6$ .

Determine the maximum and minimum values of  $f$ .

7

**Generic Scheme**

**Illustrative Scheme**

3

- <sup>1</sup> ss start to differentiate
- <sup>2</sup> ss complete derivative and set to 0
- <sup>3</sup> pd start to solve  $f'(x) = 0$
- <sup>4</sup> pd solve  $f'(x) = 0$
- <sup>5</sup> ic evaluate  $f$  at relevant stationary point
- <sup>6</sup> ss consider end-points
- <sup>7</sup> ic state max. and min. values

- <sup>1</sup> differentiate  $x^3$  or  $-2x^2$  correctly
- <sup>2</sup>  $3x^2 - 4x - 4$  } = 0 must appear at •<sup>2</sup>
- <sup>3</sup> e.g.  $(3x+2)(x-2)$  } or •<sup>3</sup> to gain •<sup>2</sup>.
- <sup>4</sup>  $-\frac{2}{3}, 2$
- <sup>5</sup>  $f(2) = -2$
- <sup>6</sup>  $f(0) = 6$  and  $f(3) = 3$
- <sup>7</sup> max. 6 and min. -2

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**Notes**

1. The only valid approach is via differentiation. A numerical approach can only gain •<sup>6</sup>.
2. Candidates who consider stationary points only cannot gain •<sup>6</sup> or •<sup>7</sup>.
3. Treat maximum (0, 6) and minimum (2, -2) as bad form.
4. Cross marking is **not** applicable to •<sup>6</sup> or •<sup>7</sup>.
5. Ignore any nature table which may appear in a candidate's solution, however (2, -2) at table is sufficient for •<sup>5</sup>.

**Regularly occurring responses**

**Response 1 : Algebraic issues in working**

**Candidate A**

$$y' = 3x^2 - 4x - 4 \quad \checkmark$$

$$(3x-2)(x+2) \quad \times$$

$$x = \frac{2}{3}, \quad x = -2 \quad \checkmark$$

$$\text{When } x = \frac{2}{3}, \quad y = \frac{74}{27} \quad \checkmark$$

$$f(0) = 6 \text{ and } f(3) = 3 \quad \checkmark$$

$$\text{max} = 6, \quad \text{min} = 2\frac{20}{27} \quad \checkmark$$

- <sup>1</sup> ✓
- <sup>2</sup> ✗
- <sup>3</sup> ✗
- <sup>4</sup> ✗
- <sup>5</sup> ✗
- <sup>6</sup> ✓
- <sup>7</sup> ✓

**Candidate B**

$$3x^2 - 4x - 4 = 0 \quad \checkmark$$

$$(3x-2)(x-2) \quad \times$$

$$x = \frac{2}{3} \text{ or } x = 2 \quad \checkmark$$

$$\text{so } f(2) = -2 \quad \times$$

- <sup>1</sup> ✓
- <sup>2</sup> ✓
- <sup>3</sup> ✗
- <sup>4</sup> ✗
- <sup>5</sup> ✗

Since  $\frac{2}{3}$  is within the domain,  $f\left(\frac{2}{3}\right)$  must also be calculated to gain •<sup>5</sup>.

**Candidate C**

$$3x^2 - 4x - 4 \quad \checkmark$$

$$(3x+2)(x-2) \quad \checkmark$$

$$3x+2=0 \quad x-2=0$$

$$x = -\frac{2}{3} \quad x = 2 \quad \checkmark$$

$$f(2) = -2 \quad \checkmark$$

- <sup>1</sup> ✓
- <sup>2</sup> ✗
- <sup>3</sup> ✓
- <sup>4</sup> ✓
- <sup>5</sup> ✓

Ignore the value of  $f\left(-\frac{2}{3}\right)$  here, if it is included.

**Response 2 : Derivative not explicitly set to zero**

**Candidate D**

$$f'(x) = 3x^2 - 4x - 4 \quad \checkmark \bullet^1$$

$$f'(x) = 0 \quad \checkmark \bullet^2$$

**Candidate E**

$$f'(x) = 0 \quad \bullet^1$$

$$f'(x) = 3x^2 - 4x - 4 \quad \checkmark \bullet^2$$

$$= (3x+2)(x-2) \quad \checkmark \bullet^3$$

**Candidate F**

$$f'(x) = 0 \quad \bullet^1$$

$$3x^2 - 4x - 4 \quad \times \bullet^2$$

$$= (3x+2)(x-2) \quad \checkmark \bullet^3$$

**Candidate G**

$$f'(x) = 0 \text{ only} \quad \times \bullet^1$$

$$\quad \times \bullet^2$$

Regularly occurring responses

Response 3 : Solving quadratic equation

Candidate H

$$f'(x) = 3x^2 - 4x - 4 \quad \bullet^1 \checkmark$$

$$3x^2 - 4x - 4 = 0 \quad \bullet^2 \checkmark$$

$$3x^2 - 4x = 4 \quad \bullet^3 \times$$

$$x(3x - 4) = 4 \quad \bullet^4 \times$$

$$x = 4, \frac{4}{3} \quad \bullet^5 \times$$

Candidate I

$$3x^2 - 4x - 4 = 0 \quad \bullet^1 \checkmark \quad \bullet^2 \checkmark$$

$$x = \frac{-(-4) \pm \sqrt{(4)^2 - 4 \times 3 \times (-4)}}{2 \times 3} \quad \bullet^3 \checkmark$$

Ignore omission of negative sign at square here.

Due to 'method' chosen  $\bullet^3, \bullet^4, \bullet^5$  and  $\bullet^7$  are not available.

Response 4 : Numerical approach

Candidate J

$$f(0) = 6$$

$$f(3) = 3 \quad \bullet^6 \checkmark$$

This candidate has stayed within the interval  $0 \leq x \leq 3$ .

Candidate K

$$f(0) = 6$$

$$f(1) = 1$$

$$f(2) = -2 \quad \bullet^5 \times$$

$$f(3) = 3 \quad \bullet^6 \checkmark$$

Candidate L

$$f(0) = 6$$

$$f(1) = 1$$

$$f(2) = -2 \quad \bullet^5 \times$$

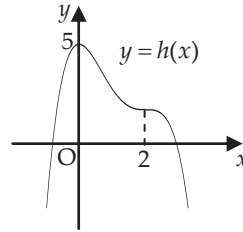
$$f(3) = 3 \quad \bullet^6 \times$$

$$f(4) = 22$$

This candidate has gone outwith the interval  $0 \leq x \leq 3$ .

For  $\bullet^5$ ,  $f(2)$  must come from calculus and not from any other approach.

- 4 The diagram below shows the graph of a quartic  $y = h(x)$ , with stationary points at  $x=0$  and  $x=2$ .



On separate diagrams sketch the graphs of:

- (a)  $y = h'(x)$ ;  
 (b)  $y = 2 - h'(x)$ .

3  
3

### Generic Scheme

### Illustrative Scheme

#### 4 (a)

- <sup>1</sup> ic identify roots
- <sup>2</sup> ic interpret point of inflection
- <sup>3</sup> ic complete cubic curve

- <sup>1</sup> 0 and 2 only
- <sup>2</sup> turning point at (2, 0)
- <sup>3</sup> cubic, passing through O with negative gradient

3

#### Notes

1. All graphs must include both the  $x$  and  $y$  axes (labelled or unlabelled), however the origin need not be labelled.
2. No marks are available unless a graph is attempted.
3. No marks are available to a candidate who makes several attempts at a graph on the same diagram, unless it is clear which is the final graph.
4. A linear graph gains no marks in both (a) and (b).

#### 4 (b)

- <sup>4</sup> ic reflection in  $x$ -axis
- <sup>5</sup> ic translation  $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$
- <sup>6</sup> ic annotation of 'transformed' graph

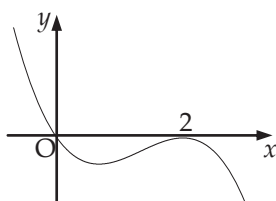
- <sup>4</sup> reflection of graph in (a) in  $x$ -axis
- <sup>5</sup> graph moves parallel to  $y$ -axis by 2 units upwards
- <sup>6</sup> two 'transformed' points appropriately annotated (see Note 5)

3

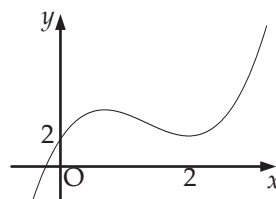
#### Notes

5. 'Transformed' here means a reflection followed by a translation.
6. •<sup>4</sup> and •<sup>5</sup> apply to the entire curve.
7. In each of the following circumstances :
  - Candidates who transform the original graph
  - Candidates who sketch a parabola in (a)
 mark the candidate's attempt as normal and unless a mark of 0 has been scored, deduct the last mark awarded. Indicate this with ✘ (see Regular occurring response G).
8. A reflection in any line parallel to the  $y$ -axis does not gain •<sup>4</sup> or •<sup>6</sup>.
9. A translation other than  $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$  does not gain •<sup>5</sup> or •<sup>6</sup>.

Graph for (a)



Graph for (b)

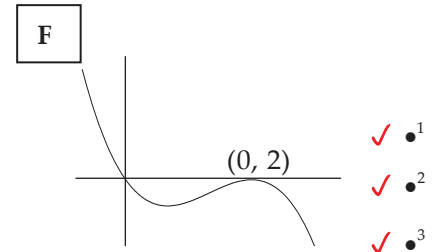
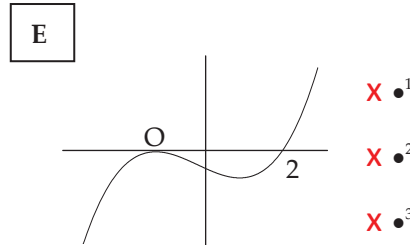
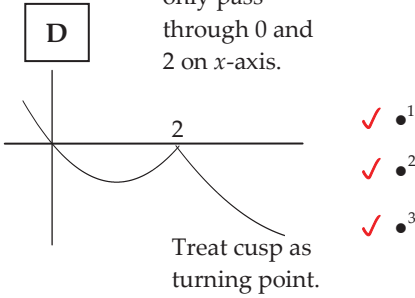
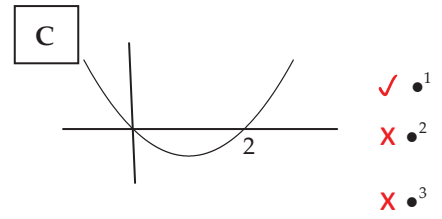
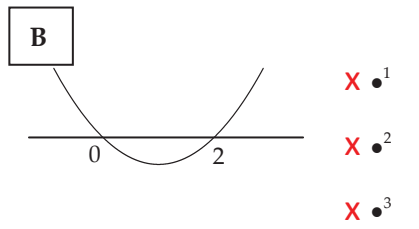
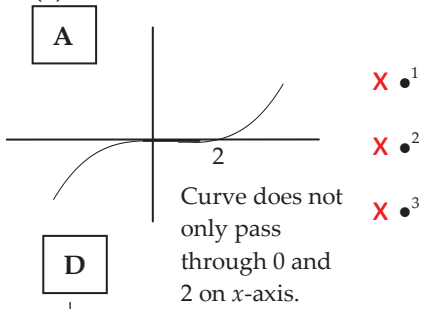


- <sup>4</sup> ic reflection in  $x$ -axis
- <sup>5</sup> ic translation  $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

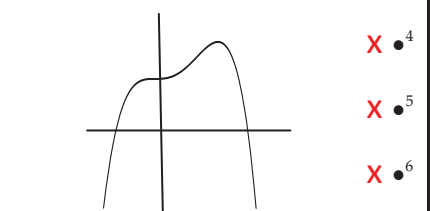
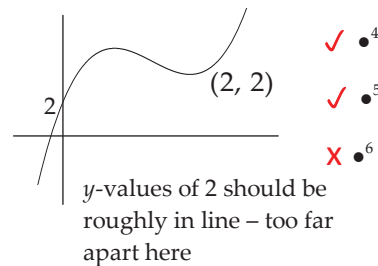
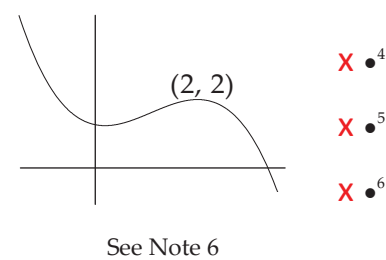
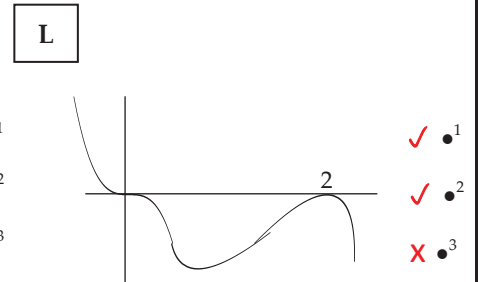
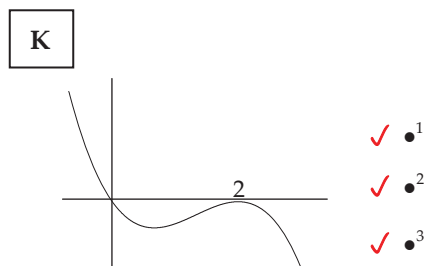
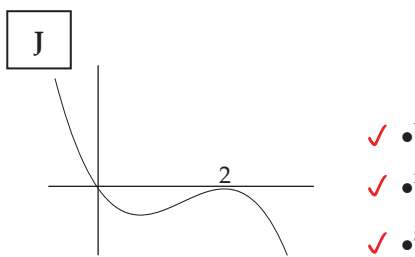
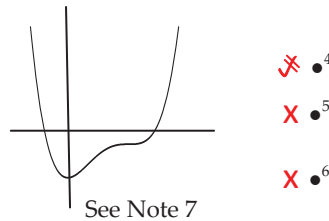
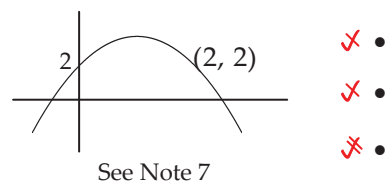
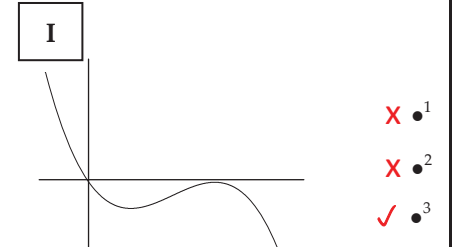
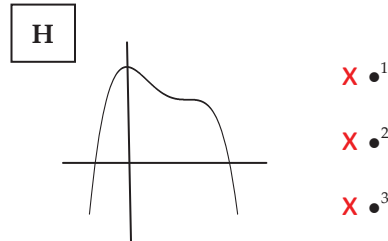
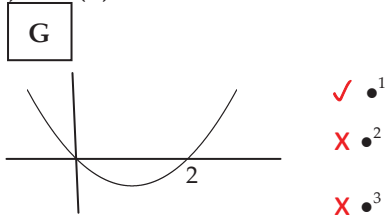
4 continued

Regularly occurring responses

In (a)



In (a) and (b)



5 A is the point (3, -3, 0), B is (2, -3, 1) and C is (4, k, 0).

(a) (i) Express  $\overline{BA}$  and  $\overline{BC}$  in component form.

(ii) Show that  $\cos \hat{ABC} = \frac{3}{\sqrt{2(k^2 + 6k + 14)}}$ .

7

**Generic Scheme**

**Illustrative Scheme**

5(a)

•<sup>1</sup> ic interpret vector

•<sup>2</sup> pd process vector

•<sup>3</sup> ss use scalar product

•<sup>4</sup> pd find scalar product

•<sup>5</sup> pd find  $|\overline{BA}|$

•<sup>6</sup> ic find expression for  $|\overline{BC}|$

•<sup>7</sup> ic complete to result

•<sup>1</sup>  $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

Treat  $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$  written as (1, 0, -1) as bad form.

•<sup>2</sup>  $\begin{pmatrix} 2 \\ k+3 \\ -1 \end{pmatrix}$

•<sup>3</sup>  $\cos \hat{ABC} = \frac{\overline{BA} \cdot \overline{BC}}{|\overline{BA}| |\overline{BC}|}$  see Note 1

•<sup>4</sup> 3

•<sup>5</sup>  $\sqrt{2}$

•<sup>6</sup>  $\sqrt{2^2 + (k+3)^2 + (-1)^2}$  or equivalent

•<sup>7</sup>  $\frac{3}{\sqrt{2}\sqrt{k^2 + 6k + 14}}$  and  $\frac{3}{\sqrt{2(k^2 + 6k + 14)}}$

or  $|\overline{BA}| |\overline{BC}| = \sqrt{2} \times \sqrt{k^2 + 6k + 14}$  and  $\frac{3}{\sqrt{2(k^2 + 6k + 14)}}$

7

**Notes**

1. If the evidence for •<sup>3</sup> does not appear explicitly, then •<sup>3</sup> is only awarded if working for •<sup>7</sup> is attempted.
2. •<sup>7</sup> is dependent on gaining •<sup>4</sup>, •<sup>5</sup> and •<sup>6</sup>.

**Regularly occurring responses**

**Response 1 : Calculating wrong angle**

**Candidate A**

$\cos AOC = \frac{\overline{OA} \cdot \overline{OC}}{|\overline{OA}| |\overline{OC}|}$  ✗ •<sup>3</sup>

$\overline{OA} \cdot \overline{OC} = 3 \times 4 + (-3) \times k + 0 \times 0 = 12 - 3k$  ✗ •<sup>4</sup>

$|\overline{OA}| = \sqrt{18}$  ✗ •<sup>5</sup>

$|\overline{OC}| = \sqrt{16 + k^2}$  ✗ •<sup>6</sup>

$\cos ABC = \frac{12 - 3k}{\sqrt{18}\sqrt{16 + k^2}}$  ✗ •<sup>7</sup>

**Candidate B**

$\cos AOB = \frac{\overline{OA} \cdot \overline{OB}}{|\overline{OA}| |\overline{OB}|}$  ✗ •<sup>3</sup>

$\overline{OA} \cdot \overline{OB} = 3 \times 2 + (-3) \times (-3) + 0 \times 1 = 15$  ✗ •<sup>4</sup>

$|\overline{OA}| = \sqrt{18}$  ✗ •<sup>5</sup>

$|\overline{OB}| = \sqrt{14}$  ✗ •<sup>6</sup>

$\cos ABC = \frac{15}{\sqrt{18}\sqrt{14}}$  ✗ •<sup>7</sup>

## Generic Scheme

## Illustrative Scheme

5(b)

Method 1 : Squaring first

- <sup>8</sup> ic link with (a)
- <sup>9</sup> ss square both sides
- <sup>10</sup> pd rearrange into 'non-fractional' format
- <sup>11</sup> pd write in standard form
- <sup>12</sup> pd solve for  $k$

Method 2 : Dealing with fractions first

- <sup>8</sup> ic link with (a)
- <sup>9</sup> pd rearrange into 'non-fractional' format
- <sup>10</sup> ss square both sides
- <sup>11</sup> pd write in standard form
- <sup>12</sup> pd solve for  $k$

Method 1 : Squaring first

- <sup>8</sup>  $\frac{3}{\sqrt{2(k^2 + 6k + 14)}} = \cos 30^\circ$
- <sup>9</sup>  $\left(\frac{3}{\sqrt{2(k^2 + 6k + 14)}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$
- <sup>10</sup>  $k^2 + 6k + 14 = 6$  or equivalent
- <sup>11</sup>  $k^2 + 6k + 8 = 0$  or equivalent
- <sup>12</sup>  $k = -2$  or  $-4$

= 0 must appear at this stage.

Method 2 : Dealing with fractions first

- <sup>8</sup>  $\frac{3}{\sqrt{2(k^2 + 6k + 14)}} = \cos 30^\circ$
- <sup>9</sup>  $\sqrt{3}\sqrt{2(k^2 + 6k + 14)} = 6$
- <sup>10</sup>  $6(k^2 + 6k + 14) = 36$
- <sup>11</sup>  $k^2 + 6k + 8 = 0$  or equivalent
- <sup>12</sup>  $k = -2$  or  $-4$

= 0 must appear at this stage.

5

## Notes

3. The evidence for •<sup>9</sup> may appear in the working for •<sup>10</sup> in both methods.
4. •<sup>9</sup> is the only mark available to candidates who replace  $\cos 30^\circ$  by 30 in method 1 and •<sup>10</sup> in method 2.
5. All 5 marks are available to candidates who use 0.87 for  $\cos 30^\circ$  but 0.9 can gain a maximum of 4 marks.

## Regularly occurring responses

Response 2 : Working with  $\cos 30^\circ$  throughout the question

Candidate C (Method 1)

$$\cos 30^\circ = \frac{3}{\sqrt{2(k^2 + 6k + 14)}} \quad \checkmark \bullet^8$$

$$(\cos 30^\circ)^2 = \left(\frac{3}{\sqrt{2(k^2 + 6k + 14)}}\right)^2 \quad \checkmark \bullet^9$$

$$(\cos 30^\circ)^2 = \frac{9}{2(k^2 + 6k + 14)}$$

$$2(\cos 30^\circ)^2(k^2 + 6k + 14) = 9 \quad \checkmark \bullet^{10}$$

If  $\cos 30^\circ$  is subsequently evaluated then •<sup>11</sup> and •<sup>12</sup> may still be available.Response 3 : Using the wrong value for  $\cos 30^\circ$ 

Candidate D (Method 2)

$$\frac{3}{\sqrt{2(k^2 + 6k + 14)}} = \frac{1}{2} \quad \times \bullet^8$$

$$\sqrt{2(k^2 + 6k + 14)} = 6 \quad \times \bullet^9$$

$$2(k^2 + 6k + 14) = 36 \quad \times \bullet^{10}$$

$$k^2 + 6k + 14 = 18$$

$$k^2 + 6k - 4 = 0 \quad \times \bullet^{11}$$

$$k = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times (-4)}}{2 \times 1}$$

$$= 0.61, -6.61 \quad \times \bullet^{12}$$

6 For  $0 < x < \frac{\pi}{2}$ , sequences can be generated using the recurrence relation

$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(a) Why do these sequences have a limit?

2

### Generic Scheme

### Illustrative Scheme

6 (a)

- |                                                                                                                                                                                |                                                                                                                                                                          |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> <li>•<sup>1</sup> ic condition on <math>u_n</math> coefficient</li> <li>•<sup>2</sup> ic connect coefficient with given interval</li> </ul> | <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>-1 &lt; \sin x &lt; 1</math></li> <li>•<sup>2</sup> in interval, <math>0 &lt; \sin x &lt; 1</math></li> </ul> |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

2

#### Notes

1. For •<sup>1</sup> **do not** accept:
  - $\sin x$  lies between  $-1$  and  $1$
  - $-1 < x < 1$
  - $-1 < \sin < 1$
 However, accept ' $\sin x$  greater than  $-1$  and less than  $1$ ' for •<sup>1</sup>.
2. Do not accept  $-1 < a < 1$  for •<sup>1</sup> unless  $a$  is clearly identified as  $\sin x$ , which may not appear until (b).
3.  $0 < \sin x < 1$  and nothing else, does not gain •<sup>1</sup> but gains •<sup>2</sup>.
4.  $0 \leq \sin x \leq 1$  and nothing else, does not gain •<sup>1</sup> or •<sup>2</sup>.

#### Regularly occurring responses

**Response 1 :** Attempts at giving a reason for limit

##### Candidate A

This sequence has a limit because  $-1 < a < 1$ ,  
 $-1 < \sin x < 1$  within the domain. ✓

•<sup>1</sup> ✓  
 •<sup>2</sup> ✗

##### Candidate B

Since  $\sin x$  in this domain will always  
 be greater than  $0$  and less than  $1$ . ✓

•<sup>1</sup> ✗  
 •<sup>2</sup> ✓

##### Candidate C

$\sin \frac{\pi}{2} = 1$  and  $\sin 0 = 0$  so the multiplier  
 of  $u_n$  is between  $0$  and  $1$ , so it has a limit. ✓ ✓

•<sup>1</sup> ✗  
 •<sup>2</sup> ✗

##### Candidate D

$-1 \leq \sin x \leq 1$ ,  
 for  $0 < x < \frac{\pi}{2}$ ,  $0 < \sin x < 1$  ✓  
 so limit exists ✓

•<sup>1</sup> ✗  
 •<sup>2</sup> ✓

**Response 2 :** Minimum response for both marks

##### Candidate E

for  $0 < x < \frac{\pi}{2}$ ,  $0 < \sin x < 1$  •<sup>2</sup> ✓  
 so  $-1 < \sin x < 1$  •<sup>1</sup> ✓  
 so limit

##### Candidate F

if limit,  $-1 < \sin x < 1$  •<sup>1</sup> ✓  
 for  $0 < x < \frac{\pi}{2}$ ,  $0 < \sin x < 1$  •<sup>2</sup> ✓



- 6 (b) The limit of one particular sequence generated by this recurrence relation is  $\frac{1}{2}\sin x$ .  
Find the value(s) of  $x$ .

7

### Generic Scheme

### Illustrative Scheme

6 (b)

- |                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> <li>•<sup>3</sup> ss appropriate limit method</li> <li>•<sup>4</sup> ic substitute for limit</li> <li>•<sup>5</sup> ss use appropriate double angle formula</li> <li>•<sup>6</sup> pd express in standard form</li> <li>•<sup>7</sup> pd start to solve quadratic equation</li> <li>•<sup>8</sup> pd reduce to equations in <math>\sin x</math> only</li> <li>•<sup>9</sup> ic select valid solution</li> </ul> | <ul style="list-style-type: none"> <li>•<sup>3</sup> limit = <math>\frac{\cos 2x}{1 - \sin x}</math> or <math>l = \sin x \times l + \cos 2x</math></li> <li>•<sup>4</sup> <math>\frac{1}{2}\sin x = \frac{\cos 2x}{1 - \sin x}</math> or <math>\frac{1}{2}\sin x = \sin x \times \frac{1}{2}\sin x + \cos 2x</math><br/>(•<sup>3</sup> may be stated, or implied by •<sup>4</sup> in both methods)</li> <li>•<sup>5</sup> ... <math>1 - 2\sin^2 x</math> ...</li> <li>•<sup>6</sup> e.g. <math>3\sin^2 x + \sin x - 2</math></li> <li>•<sup>7</sup> e.g. <math>(3\sin x - 2)(\sin x + 1)</math> } = 0 must appear at •<sup>6</sup> or •<sup>7</sup><br/>to gain •<sup>6</sup>.</li> <li>•<sup>8</sup> <math>\sin x = \frac{2}{3}</math> or <math>\sin x = -1</math></li> <li>•<sup>9</sup> <math>x = 0.730</math> or outwith interval</li> </ul> |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

7

### Notes

5. •<sup>7</sup>, •<sup>8</sup> and •<sup>9</sup> are only available if a quadratic equation is obtained at •<sup>6</sup> stage.
6. Candidates may express the quadratic equation at the •<sup>6</sup> stage in the form  $3s^2 + s - 2 = 0$ . For candidates who do not solve a trigonometric quadratic equation at •<sup>7</sup>  $\sin x$  must appear explicitly to gain •<sup>8</sup>.
7. •<sup>7</sup>, •<sup>8</sup> and •<sup>9</sup> are not available to candidates who 'solve' a quadratic equation in the form  $ax^2 + bx = c$ ,  $c \neq 0$ .
8. For •<sup>9</sup> there must be one valid solution, and one solution outwith interval which is rejected.
9. •<sup>9</sup> is not available to candidates who leave their answer in degree measure.
10. Cross marking is available for •<sup>8</sup> and •<sup>9</sup>.

### Regularly occurring responses

**Response 3 :** Evidence for identification of  $a$  appearing in (b)

**Candidate G**

- (a)  $-1 < a < 1$  ✓<sup>1</sup>
- (b)  $L = \frac{b}{1-a} = \frac{\cos 2x}{1 - \sin x}$  ✓<sup>3</sup> ✓<sup>1</sup> ✓<sup>3</sup>

**Response 4 :** Error in algebra and subsequent quadratic equation solution

**Candidate H**

$$L = \frac{b}{1-a} = \frac{1}{2}\sin x$$

$$\frac{\cos 2x}{1 - \sin x} = \frac{1}{2}\sin x \quad \checkmark \bullet^3 \quad \checkmark \bullet^4$$

$$\cos 2x = -\frac{1}{2}\sin^2 x \quad \times \bullet^6$$

$$\frac{1}{2}\sin^2 x + \cos 2x = 0$$

$$\frac{1}{2}\sin^2 x + (1 - 2\sin^2 x) = 0 \quad \times \bullet^5$$

$$-\frac{3}{2}\sin^2 x + 1 = 0$$

$$\sin^2 x = \frac{2}{3} \quad \times \bullet^7$$

$$\sin x = \sqrt{\frac{2}{3}} \text{ and } \sin x = -\sqrt{\frac{2}{3}} \quad \times \bullet^8$$

$$x = 0.955, 2.186 \quad x = 4.097, 5.328 \quad \times \bullet^9$$

**Candidate I**

$$\frac{\cos 2x}{1 - \sin x} = \frac{1}{2}\sin x \quad \checkmark \bullet^3 \quad \checkmark \bullet^4$$

$$\frac{1}{2}\sin x(1 - \sin x) = \frac{1 - \sin^2 x}{2} \quad \times \bullet^5$$

$$\sin^2 x + \sin x - 2 = 0 \quad \times \bullet^6$$

$$(\sin x - 1)(\sin x + 2) = 0 \quad \times \bullet^7$$

$$\sin x = 1 \text{ and } \sin x = -2 \quad \times \bullet^8$$

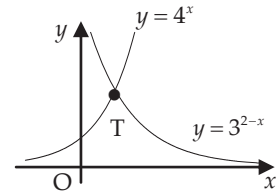
$$x = \frac{\pi}{2} \quad \text{not possible} \quad \times \bullet^9$$

See Note 8

7 The diagram shows the curves with equations  $y = 4^x$  and  $y = 3^{2-x}$ .

The graphs intersect at the point T.

(a) Show that the  $x$ -coordinate of T can be written in the form  $\frac{\log_a p}{\log_a q}$ , for all  $a > 1$ .



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### Generic Scheme

### Illustrative Scheme

7(a)

- <sup>1</sup> ss equate expressions for  $y$
- <sup>2</sup> ss take logarithms of both sides
- <sup>3</sup> ic use law of logs :  $\log_a x^n = n \log_a x$
- <sup>4</sup> pd gather like terms
- <sup>5</sup> ic use law of logs :  $\log_a p + \log_a q = \log_a pq$
- <sup>6</sup> ic complete to required form

#### Method 1

- <sup>1</sup>  $4^x = 3^{2-x}$
- <sup>2</sup>  $\log_a (4^x) = \log_a (3^{2-x})$  **stated, or implied by** •<sup>3</sup>
- <sup>3</sup>  $x \log_a 4 = (2-x) \log_a 3$
- <sup>4</sup>  $x(\log_a 4 + \log_a 3) = 2 \log_a 3$
- <sup>5</sup>  $x \log_a 12 = \log_a 9$
- <sup>6</sup>  $\frac{\log_a 9}{\log_a 12}$  **stated explicitly**

#### Method 2

- <sup>1</sup>  $4^x = 3^{2-x}$
- <sup>2</sup>  $\log_3 (4^x) = 2-x$
- <sup>3</sup>  $x \log_3 4 = 2-x$
- <sup>4</sup>  $x = \frac{2}{1 + \log_3 4}$
- <sup>5</sup>  $\frac{2 \log_3 3}{\log_3 12}$
- <sup>6</sup>  $\frac{\log_a 9}{\log_a 12}$  **stated explicitly**

#### Method 3

- <sup>1</sup>  $4^x = 3^{2-x}$
- <sup>2</sup>  $4^x = \frac{3^2}{3^x}$
- <sup>3</sup>  $12^x = 9$
- <sup>4</sup>  $\log_a 12^x = \log_a 9$
- <sup>5</sup>  $x \log_a 12 = \log_a 9$
- <sup>6</sup>  $\frac{\log_a 9}{\log_a 12}$  **stated explicitly**

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In methods 1 and 2:

If the first line of working is that at the •<sup>2</sup> stage, then •<sup>1</sup> and •<sup>2</sup> are awarded.

If the first line of working is that at the •<sup>3</sup> stage, then only •<sup>2</sup> and •<sup>3</sup> are awarded.

### Notes

1. In methods 1 and 2, if no base is indicated then •<sup>2</sup> is not available, however •<sup>3</sup>, •<sup>4</sup> and •<sup>5</sup> are still available. In method 3, if no base is indicated then •<sup>4</sup> is not available, however •<sup>5</sup> is still available.
2. In all methods, if a numerical base is used then •<sup>6</sup> is not available.
3. In method 1, the omission of brackets at the •<sup>3</sup> stage is treated as bad form, see Response 1.
4.  $p$  and  $q$  must be numerical values.

### Regularly occurring responses

**Response 1 : Omission of brackets around  $2-x$**

**Candidate A**  $4^x = 3^{2-x}$  ✓ •<sup>1</sup>  
 $x \log_a 4 = 2 - x \log_a 3$  ✓ •<sup>2</sup> ✓ •<sup>3</sup>

**Candidate B**  $4^x = 3^{2-x}$  ✓ •<sup>1</sup>  
 $x \log_a 4 = 2 - x \log_a 3$  ✓ •<sup>2</sup> ✓ •<sup>3</sup>  
 $x(\log_a 4 + \log_a 3) = 2$  ✗ •<sup>4</sup>  
 $x \log_a 12 = 2$  ✗ •<sup>5</sup>

**Response 2 : Using different bases**  
**Candidate C**

$4^x = 3^{2-x}$  ✓ •<sup>1</sup>  
 $\log_3 4^x = \log_4 3^{2-x}$  ✗ •<sup>2</sup>  
 $x \log_3 4 = (2-x) \log_4 3$  ✗ •<sup>3</sup>

**Response 3 : Taking logs first**  
**Candidate D**

$y = 4^x$  and  $y = 3^{2-x}$   
 $\log_a y = \log_a 4^x$  and  $\log_a y = \log_a 3^{2-x}$  ✓ •<sup>2</sup>  
 $\log_a y = x \log_a 4$  and  $\log_a y = (2-x) \log_a 3$  ✓ •<sup>3</sup>  
 $x \log_a 4 = (2-x) \log_a 3$  ✓ •<sup>1</sup>

$x = \frac{2}{\log_a 12}$   
 $= \frac{2 \log_a a}{\log_a 12}$   
 $= \frac{\log_a a^2}{\log_a 12}$  ✗ •<sup>6</sup>

## Generic Scheme

## Illustrative Scheme

7(b)

- <sup>7</sup> ic substitute in for  $x$
- <sup>8</sup> pd process  $y$

- <sup>7</sup> e.g.  $y = 4^{\frac{\log_a 9}{\log_a 12}}$
- <sup>8</sup> e.g.  $y \approx 4^{0.8842} \approx 3 \cdot 4$

stated, or implied by •<sup>8</sup>

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## Notes

5. Candidates must work to at least two significant figures in (b) e.g.  $4^{0.9} = 3.5$  does not gain •<sup>8</sup>, but •<sup>7</sup> is available.
6. •<sup>8</sup> is only available if the power used comes from  $\frac{\log_a p}{\log_a q}$  in (a).

## Regularly occurring responses

Response 4 : Using  $p$  and  $q$  as integer values without working

Candidate E

$$\left. \begin{array}{l} p = 4 \\ q = 3 \end{array} \right\} y = 4^{1.26} = 5.74 \text{ or } 5.75 \quad \begin{array}{l} \times \bullet^7 \\ \times \bullet^8 \end{array}$$

Candidate F

$$\left. \begin{array}{l} p = 3 \\ q = 4 \end{array} \right\} y = 4^{0.79} = 2.99 \text{ or } 3 \quad \begin{array}{l} \times \bullet^7 \\ \times \bullet^8 \end{array}$$

Response 5 : Using integer values calculated in (a)

Candidate G

$$\left. \begin{array}{l} p = 10 \\ q = 4 \end{array} \right\} y = 4^{2.5} = 32 \quad \begin{array}{l} \times \bullet^7 \\ \times \bullet^8 \end{array}$$

[END OF MARKING INSTRUCTIONS]