



2011 Mathematics

Higher

Finalised Marking Instructions

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General Comments

These marking instructions are for use with the 2011 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which you will commonly see throughout your marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general you should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

- 1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.
- 2 Award one mark for each •. There are no half marks.
- 3 Working subsequent to an error must be **followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction of mark(s) should be made.

4 Marking Symbols

No comments, words or acronyms should be written on scripts. Please use the following and **nothing else**.



A tick should be used where a piece of working is correct and gains a mark. You are not expected to tick every line of working but you must check through the whole of a response.



Where a mark is lost, the error should be underlined in **red** at the point where it first occurs, and not at any subsequent stage of the working.



A cross-tick should be used to indicate “correct” working where a mark is awarded as a result of **follow through** from an error.



A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been **eased**.



A tilde should be used to indicate a minor transgression which is not being penalised, e.g. **bad form**.



This should be used where a candidate is given the **benefit of the doubt**.



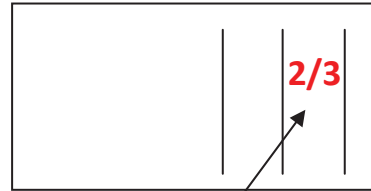
A roof should be used to show that something is missing, such as a crucial step in the working or part of a solution.

These will help you to maintain consistency in your marking and will assist the examiners in the later stages of SQA procedures.

- 5 Regularly Occurring Responses (ROR) are shown on the marking scheme to help mark common solutions that are non-routine.
- 6 RORs may also be used as a guide in marking other non-routine candidate responses.
- 7 The mark for **each part** of a question should be entered in **red** in the **outer** right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, **as a single number**, should be written.



Marks in this column - single numbers only



Do not record marks on scripts in this manner.

- 8 Where a candidate has scored zero for any question, or part of a question, 0 should be written in the right hand margin beside their answer.
- 9 Every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol, should have a tick placed in the bottom right hand margin.
- 10 Where a solution is spread over several pages the marks should be recorded at the end of the solution. This should be indicated with a down arrow (\Downarrow), in the margin, at the earlier stages.

The examples below illustrate the use of the marking symbols .

Example 1

$$y = x^3 - 6x^2 \quad \bullet^1 \checkmark$$

$$\frac{dy}{dx} = 3x^2 - 12 \quad \checkmark \bullet^1 \times \bullet^2 \quad \bullet^2 \times$$

$$3x^2 - 12 = 0 \quad \times \bullet^3 \quad \bullet^3 \times$$

$$x = 2 \quad \wedge \bullet^4 \quad \bullet^4 \wedge$$

$$y = -16 \quad \times \bullet^5 \quad \bullet^5 \times$$

Example 2

$$A(4,4,0), B(2,2,6), C(2,2,0)$$

$$\overline{AB} = \mathbf{b} + \mathbf{a} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \quad \times \bullet^1$$

$$\overline{AC} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} \quad \times \bullet^2 \text{ (repeated error)}$$

Example 3

$$3 \sin x - 5 \cos x$$

$$k \sin x \cos a - \cos x \sin a \quad \checkmark \bullet^1$$

$$k \cos a = 3, k \sin a = 5 \quad \checkmark \bullet^2$$

Example 4

Find intersection of $x + 3y = 23$ and $y = 3x - 9$

$$y - 3x = 9 \quad \text{Strategy mark awarded.}$$

$$3y + x = 24 \quad \checkmark \bullet^1 \text{ (despite two errors)}$$

$$3y - 9x = 27$$

$$x = -\frac{3}{10} \quad \times \bullet^2 \text{ The subsequent pd mark is lost (Note 12)}$$

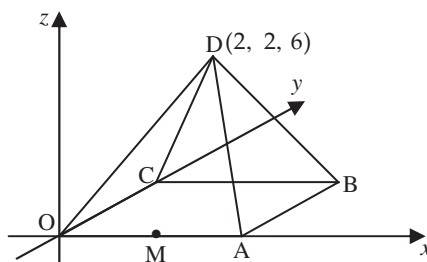
- 11** Where a transcription error (paper to script or within script) occurs, a mark is lost.
e.g.

This is a transcription error and so mark is lost.	$x^2 + 5x + 7 = 9x + 4$ ✓ $x - 4x + 3 = 0$ ✗ $x = 1$ ✗
Eased as not solution of a quadratic equation.	
Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.	$x^2 + 5x + 7 = 9x + 4$ ✓ $x - 4x + 3 = 0$ ✓ $(x-3)(x-1) = 0$ ✓ $x = 1 \text{ or } 3$ ✓

- 12** In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6 \times 6 = 12$, candidates lose the opportunity of gaining the appropriate *ic* or *pd* mark.
- 13** A processing error made at a strategy mark stage is penalised at the next *pd* or *ic* mark available within that part of the question. The strategy mark may still be awarded.
- 14** As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking scheme, a correct answer with no working receives no credit.
- 15** Unless specifically mentioned in the marking scheme, do not penalise:
- Working subsequent to a **correct** answer;
 - Correct working in the wrong part of a question;
 - Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
 - Omission of units;
 - Bad form.
- 16** No piece of working should be ignored without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme. Answers which are widely off-beam are unlikely to include anything of relevance, but candidates may still have the opportunity of gaining the odd mark or two, provided it satisfies the criteria for the marks.
- 17** In the exceptional circumstance where you are in doubt whether a mark should or should not be awarded, err on the generous side and award the mark.

- 18** Scored out or erased working which **has not been replaced** should be marked where still legible. However, if the scored out or erased working **has been replaced**, only the work which has not been scored out should be marked.
- 19** A valid approach, within Mathematical problem solving, is to try different strategies. Where this occurs, all working should be marked. The mark awarded to the candidate is from the *highest* scoring strategy. This is distinctly different from the candidate who gives two or more solutions to a question/part of a question, deliberately leaving all solutions, hoping to gain some benefit. All such contradictory responses should be marked and the *lowest* mark given.
- 20** It is of great importance that the utmost care should be exercised in adding up the marks. The recommended procedure is as follows:
- Step 1 Manually calculate the total from the candidate's script.
 - Step 2 Check this total using the grid issued with these marking instructions.
 - Step 3 Input the scores and obtain confirmation of your total from the EMC screen.
(This should highlight any discrepancies hitherto undiscovered.)
- 21** Place the candidate's script for Paper 2 inside the script for Paper 1 and write the candidate's total score (i.e. Paper 1 Section B + Paper 2) in the space provided on the front cover of the script for Paper 1.
- 22** In cases of difficulty, covered neither in detail nor in principle in these instructions, contact your Team Leader (TL) in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with your TL. Please see the General Marking Instructions for PA Referrals.

- 1 D,OABC is a square based pyramid as shown in the diagram below.
 O is the origin, D is the point (2, 2, 6) and OA = 4 units.
 M is the mid-point of OA.
 (a) State the coordinates of B.
 (b) Express \overrightarrow{DB} and \overrightarrow{DM} in component form.



1
3

Throughout this question, coordinates written as components and vice versa are treated as bad form.

Generic Scheme

Illustrative Scheme

1 (a)

•¹ ic state coordinates of B

•¹ (4, 4, 0)

1

1 (b)

•² pd state components of \overrightarrow{DB}

•² $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$

•³ ic state coordinates of M

•³ (2, 0, 0) stated, or implied by •⁴

•⁴ pd state components of \overrightarrow{DM}

•⁴ $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$

3

Regularly occurring responses

Response 1A (Transcription error for D)

$$\overrightarrow{DB} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix} \quad \times \bullet^2$$

$$\overrightarrow{DM} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -6 \end{pmatrix} \quad \checkmark \bullet^3 \quad \times \bullet^4$$

2 marks out of 3

Response 1B (Transcription error for D)

$$\overrightarrow{DB} = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix} \text{ and } \overrightarrow{DM} = \begin{pmatrix} 0 \\ -6 \\ -6 \end{pmatrix} \text{ with no working.}$$

0 marks out of 3

Response 2A

$$\overrightarrow{DB} = \underline{\mathbf{d}} + \mathbf{b} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \quad \times \bullet^2$$

$$\overrightarrow{DM} = \mathbf{d} + \mathbf{m} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} \quad \checkmark \bullet^3 \quad \times \bullet^4$$

2 marks out of 3

Response 2B

$$\overrightarrow{DB} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \text{ and } \overrightarrow{DM} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} \text{ with no working.}$$

0 marks out of 3

1 (c) Find the size of angle BDM.

5

Generic Scheme

Illustrative Scheme

1 (c)

- ⁵ ss know to use scalar product
- ⁶ pd find scalar product
- ⁷ pd find magnitude of a vector
- ⁸ pd find magnitude of a vector
- ⁹ pd evaluate angle BDM

- ⁵ $\cos \hat{BDM} = \frac{\overline{DB} \cdot \overline{DM}}{|\overline{DB}| |\overline{DM}|}$ stated, or implied by •⁹
- ⁶ $\overline{DB} \cdot \overline{DM} = 32$
- ⁷ $|\overline{DB}| = \sqrt{44}$
- ⁸ $|\overline{DM}| = \sqrt{40}$
- ⁹ $40 \cdot 3^\circ$ or 0.703 rads

5

Notes

- ⁵ is not available to candidates who evaluate the wrong angle.
- If candidates do not attempt •⁹, then •⁵ is only available if the formula quoted relates to the labelling in the question.
- ⁹ should be awarded to any answer which rounds to 40° or 0.7 rads.
- In the event that both magnitudes are equal or there is only one non-zero component, •⁸ is not available.

Regularly occurring responses

Response 3A

$$\cos \hat{BOM} = \frac{\overline{OB} \cdot \overline{OM}}{|\overline{OB}| |\overline{OM}|} \times \bullet^5$$

$$\overline{OB} \cdot \overline{OM} = 8 \times \bullet^6$$

$$|\overline{OB}| = \sqrt{32} \times \bullet^7$$

$$|\overline{OM}| = 2 \times \bullet^8$$

$$45^\circ \times \bullet^9$$

3 marks out of 5

Response 3B

$$\cos \hat{BOD} = \frac{\overline{OB} \cdot \overline{OD}}{|\overline{OB}| |\overline{OD}|} \times \bullet^5$$

$$\overline{OB} \cdot \overline{OD} = 16 \times \bullet^6$$

$$|\overline{OB}| = \sqrt{32} \times \bullet^7$$

$$|\overline{OD}| = \sqrt{44} \times \bullet^8$$

$$64 \cdot 8^\circ \times \bullet^9$$

4 marks out of 5

Response 3C

$$\cos \hat{DBM} = \frac{\overline{BD} \cdot \overline{BM}}{|\overline{BD}| |\overline{BM}|} \times \bullet^5$$

$$\overline{BD} \cdot \overline{BM} = 12 \times \bullet^6$$

$$|\overline{BD}| = \sqrt{44} \times \bullet^7$$

$$|\overline{BM}| = \sqrt{20} \times \bullet^8$$

$$66 \cdot 1^\circ \times \bullet^9$$

4 marks out of 5

Response 4

$$\cos \hat{BDM} = \frac{\overline{BD} \cdot \overline{DM}}{|\overline{BD}| |\overline{DM}|} \times \bullet^5$$

$$\overline{BD} \cdot \overline{DM} = -32 \times \bullet^6$$

$$|\overline{BD}| = \sqrt{44} \times \bullet^7$$

$$|\overline{DM}| = \sqrt{40} \times \bullet^8$$

$$139 \cdot 7^\circ \times \bullet^9$$

4 marks out of 5

Response 5A

(Scalar Product is 0)

$$\text{In 1(b)} \overline{DB} = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}, \overline{DM} = \begin{pmatrix} 0 \\ 6 \\ -2 \end{pmatrix}$$

$$\cos \hat{BDM} = \frac{\overline{DB} \cdot \overline{DM}}{|\overline{DB}| |\overline{DM}|} \checkmark \bullet^5$$

$$\overline{DB} \cdot \overline{DM} = 0 \times \bullet^6$$

$$|\overline{DB}| = \sqrt{44} \checkmark \bullet^7$$

$$|\overline{DM}| = \sqrt{40} \checkmark \bullet^8$$

$$90^\circ \times \bullet^9$$

5 marks out of 5

Response 5B

$$\text{In 1(b)} \overline{DB} = \begin{pmatrix} 2 \\ -2 \\ -6 \end{pmatrix}, \overline{DM} = \begin{pmatrix} 0 \\ 6 \\ -2 \end{pmatrix}$$

$$\cos \hat{BDM} = \frac{\overline{DB} \cdot \overline{DM}}{|\overline{DB}| |\overline{DM}|} \checkmark \bullet^5$$

$$\overline{DB} \cdot \overline{DM} = 0 \times \bullet^6$$

so perpendicular $\times \bullet^9$

$$\wedge \bullet^7 \wedge \bullet^8$$

3 marks out of 5

Response 6 (Cosine rule)

$$\cos \hat{BDM} = \frac{DB^2 + DM^2 - BM^2}{2 \times DB \times DM} \checkmark \bullet^5$$

$$DB = \sqrt{44} \checkmark \bullet^6$$

$$DM = \sqrt{40} \checkmark \bullet^7$$

$$BM = \sqrt{20} \checkmark \bullet^8$$

$$40 \cdot 3^\circ \checkmark \bullet^9$$

5 marks out of 5

2	Functions f, g and h are defined on the set of real numbers by <ul style="list-style-type: none"> • $f(x) = x^3 - 1$ • $g(x) = 3x + 1$ • $h(x) = 4x - 5$ 	
(a)	Find $g(f(x))$.	2
(b)	Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$.	1

Generic Scheme

Illustrative Scheme

<p>2 (a)</p> <ul style="list-style-type: none"> •¹ ic interpret notation •² ic complete process 	<ul style="list-style-type: none"> •¹ $g(x^3 - 1)$ stated, or implied by •² •² $3(x^3 - 1) + 1$
--	--

Notes

- $3x^3 - 2$ without working gains only 1 mark.
- $f(g(x))$ loses •¹ but will gain •² for $(3x+1)^3 - 1$.
- $f(x) \times g(x)$ loses both •¹ and •².

2

<p>2 (b)</p> <ul style="list-style-type: none"> •³ ic substitute and complete 	<ul style="list-style-type: none"> •³ $3(x^3 - 1) + 1 + x(4x - 5)$ $= 3x^3 + 4x^2 - 5x - 2$ stated explicitly or $3(x^3 - 1) + 1 + 4x^2 - 5x$ $= 3x^3 + 4x^2 - 5x - 2$ stated explicitly or $3x^3 - 2 + x(4x - 5)$ $= 3x^3 + 4x^2 - 5x - 2$ stated explicitly or $3x^3 - 2 + 4x^2 - 5x$ $= 3x^3 + 4x^2 - 5x - 2$ stated explicitly
--	---

1

Regularly occurring responses

CAVE : Watch out for erroneous working leading to the required cubic.

Response 1 $3x^3 - 2 + x(4x + 5) = 3x^3 + 4x^2 - 5x - 2$ ✗ •³

Response 2 $3x^3 - 4 + x(4x - 5) = 3x^3 + 4x^2 - 5x - 2$ ✗ •³

Response 3 From (a) $(3x+1)^3 - 1$
 In (b) $3x^3 + 3 - 1 + x(4x - 5) = 3x^3 + 2 + 4x^2 - 5x$ ✗ •³
 $= 3x^3 + 4x^2 - 5x - 2$

Response 4A From (a) $g(f(x)) = 3x^3 - 2$
 In (b) $xh(x) = 4x^2 - 5x$
 $\wedge 3x^3 + 4x^2 - 5x - 2$ ✗ •³

Response 4B From (a) $g(f(x)) = 3x^3 - 2$
 In (b) $3x^2 - 2 + 4x^2 - 5x$ ✗ •³
 $= 3x^3 + 4x^2 - 5x - 2$

Note : •³ is not available to candidates who leave their answer as $3x^3 - 2 + 4x^2 - 5x$.

As the form of the answer was given in the question, this mark is not available.

2	(c)	(i) Show that $(x-1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.	
		(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.	5
	(d)	Hence solve $g(f(x)) + xh(x) = 0$.	1

Generic Scheme

Illustrative Scheme

2 (c)		Method 1 : Using synthetic division	
• ⁴ ss	know to use $x = 1$	• ⁴ 1 3 4 -5 -2	<div style="border: 1px solid black; padding: 5px;"> If only the word 'factor' appears, it must be linked to the 0 in the table. The link could be as little as 'so', '∴', '→', '⇒' or 'hence'. The word 'factor' only, with no link, does not gain •⁶. </div>
• ⁵ pd	complete evaluation	• ⁵ 1 3 4 -5 -2 3 7 2	
• ⁶ ic	state conclusion	• ⁶ "remainder is zero so $(x-1)$ is a factor", accept " $(x-1)$ is a factor"	
• ⁷ ic	find quadratic factor	• ⁷ $3x^2 + 7x + 2$	stated, or implied by •⁸
• ⁸ pd	factorise completely	• ⁸ $(x-1)(3x+1)(x+2)$	stated explicitly
		Method 2 : Using substitution and inspection	
		• ⁴ know to use $x = 1$	
		• ⁵ $3 + 4 - 5 - 2 = 0$	
		• ⁶ $(x-1)$ is a factor	
		• ⁷ $(x-1)(3x^2 + 7x + 2)$	stated, or implied by •⁸
		• ⁸ $(x-1)(3x+1)(x+2)$	stated explicitly
5			
Notes			
4. • ⁶ is only available as a consequence of the evidence for • ⁴ and • ⁵ .			
5. Communication at • ⁶ must be consistent with working at • ⁵ . i.e. candidate's working must arrive legitimately at zero before • ⁶ is awarded. If the remainder is not 0 then an appropriate statement would be ' $(x-1)$ is not a factor'.			
6. Unacceptable statements : $x = 1$ is a factor, $(x+1)$ is a factor, $x = 1$ is a root, $(x-1)$ is a root etc.			
7. • ⁹ cannot be awarded for solving $3x^3 + 4x^2 - 5x - 2 = 0$ in (c).			

2 (d)	• ⁹ ic	interpret and solve equation in (d) • ⁹ $-2, -\frac{1}{3}$ and 1	<div style="border: 1px solid black; padding: 5px;"> These must appear explicitly here at (d). </div>	1
Notes				
8. From (c) $(x-1)(3x+1)(x+2)$ leading to $x = 1, x = -\frac{1}{3}$ and $x = -2$ then $(1, 0), (-\frac{1}{3}, 0)$ and $(-2, 0)$ gains • ⁹ . However, $(x-1)(3x+1)(x+2)$ leading to $(1, 0), (-\frac{1}{3}, 0)$ and $(-2, 0)$ only does not gain • ⁹ .				
9. From (c) $(3x+1)(x+2)$ only, leading to $x = -\frac{1}{3}, x = -2$ does not gain • ⁹ as equation solved is not a cubic.				

3	(a)	A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$. Write down the values of u_1 and u_2 .	1
	(b)	A second sequence is given by 4, 5, 7, 11, It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$. Find the values of p and q .	3

Generic Scheme

Illustrative Scheme

3 (a)	
<ul style="list-style-type: none"> •¹ pd find terms of sequence 	<ul style="list-style-type: none"> •¹ $u_1 = 8$ and $u_2 = -4$ Accept "8 and -4"
1	

3 (b)	
<ul style="list-style-type: none"> •² ic interpret sequence •³ ss solve for one variable •⁴ pd state second variable 	<ul style="list-style-type: none"> •² e.g. $4p + q = 5$ and $5p + q = 7$ •³ $p = 2$ or $q = -3$ •⁴ $q = -3$ or $p = 2$
3	

Notes

1. Candidates may use $7p + q = 11$ as one of their equations at •².
2. Treat equations like $p4 + q = 5$ or $p(4) + q = 5$ as bad form.
3. Candidates should not be penalised for using $u_{n+1} = pu_n + q$.

Regularly occurring responses

<p>Response 1A (No working)</p> <p>$p = 2$ and $q = -3$</p> <p>or $v_{n+1} = 2v_n - 3$</p>	<p>Response 1B (Only one equation)</p> <p>$4p + q = 5$</p> <p>$p = 2$ and $q = -3$</p>	<p>Response 1C (By verification)</p> <p>$p = 2$ and $q = -3$ (ex nihilo)</p> <p>$v_2 = 8 - 3 = 5$</p> <p>and $v_3 = 10 - 3 = 7$</p>
1 mark out of 3	1 mark out of 3	2 marks out of 3

- 3 (c) Either the sequence in (a) or the sequence in (b) has a limit.
- (i) Calculate this limit.
- (ii) Why does the other sequence not have a limit?

3

Generic Scheme

Illustrative Scheme

3 (c)

- ⁵ ss know how to find valid limit
- ⁶ pd calculate a valid limit only
- ⁷ ic state reason

- ⁵ $l = -\frac{1}{2}l$ or $l = \frac{0}{1 - (-\frac{1}{2})}$
- ⁶ $l = 0$
- ⁷ outside interval $-1 < p < 1$

3

Notes

4. Just stating that $l = al + b$ or $l = \frac{b}{1-a}$ is not sufficient for •⁵.
5. Any calculations based on formulae masquerading as a limit rule cannot gain •⁵ and •⁶.
6. For candidates who use 'b = 0', •⁶ is only available to those who simplify $\frac{0}{\dots}$ to 0.
7. Accept $2 > 1$ or $p > 1$ for •⁷. This may be expressed in words.
8. Candidates who use a without reference to p or 2 cannot gain •⁷.

Using $l = \frac{b}{1-a}$,
 $a = -\frac{1}{2}$ and
 $b = 0$, without
 substituting,
 and stating
 $l = 0$, gains •⁵
 but not •⁶.

Regularly occurring responses

Response 2

$l = \frac{0}{\frac{1}{2}}$ ✗ •⁵
 $l = 0$ ✗ •⁶
 See note 5

0 marks out of 3

Response 3A

From (b) $p = \frac{3}{4}$ and $q = 2$
 In (c) $l = \frac{3}{4}l + 2$ and $l = -\frac{1}{2}l$ ✓ •⁵
 $l = 8$ $l = 0$ ✓ •⁶
 Both limits exist as $-1 < \frac{3}{4} < 1$ and $-1 < -\frac{1}{2} < 1$ ✓ •⁷

3 marks out of 3

Response 3B

From (b) $p = \frac{3}{4}$ and $q = 2$
 In (c) $l = \frac{3}{4}l + 2$ and $l = -\frac{1}{2}l$ ✓ •⁵
 $l = 8$ $l = 0$ ✓ •⁶
 Impossible ✗ •⁷

2 marks out of 3

Response 4A

$l = \frac{0}{1 - (-\frac{1}{2})}$ ✓ •⁵ and $l = \frac{-3}{1-2}$
 $l = 0$ $l = 3$ ✗ •⁶
 First has limit because $-1 < -\frac{1}{2} < 1$ ✗ •⁷

1 mark out of 3

Response 4B

$l = \frac{0}{1 - (-\frac{1}{2})}$ ✓ •⁵ and $l = \frac{-3}{1-2}$
 $l = 0$ $l = 3$ ✗ •⁶
 Second sequence has no limit as $-1 < 2 < 1$ not true ✓ •⁷

2 marks out of 3

Response 5A

$l = -\frac{1}{2}l$ and $l = 2l - 3$ ✓ •⁵
 $l = 0$ $l = 3$ ✓ •⁶
 1st has limit because $-1 < 0 < 1$ ✗ •⁷

2 marks out of 3

Response 5B

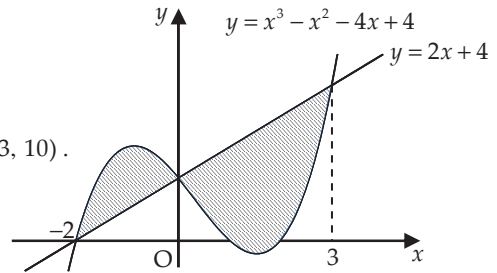
$l = -\frac{1}{2}l$ and $l = 2l - 3$ ✓ •⁵
 $l = 0$ $l = 3$ ✓ •⁶
 Second sequence has no limit because 2 is not between ✓ •⁷
 -1 and 1

3 marks out of 3

4 The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.

Calculate the total shaded area.



10

Generic Scheme

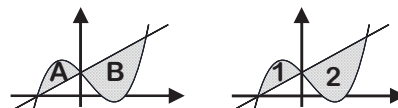
Illustrative Scheme

4		
• ¹ ss	know to integrate	• ¹ $\int \dots$ or attempt integration
• ² ic	know to deal with areas on each side of y -axis	• ² Evidence of attempting to interpret the diagram to left of y -axis separately from diagram to the right.
• ³ ic	interpret limits of one area	• ³ \int_{-2}^0 e.g. \int_0^3 with no other
• ⁴ ic	use "upper - lower"	• ⁴ $(x^3 - x^2 - 4x + 4) - (2x + 4)$ $(2x + 4) - (x^3 - x^2 - 4x + 4)$
• ⁵ pd	integrate	• ⁵ $\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$ $3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
• ⁶ ic	substitute in limits	• ⁶ $-\left(\frac{1}{4}(-2)^4 - \frac{1}{3}(-2)^3 - 3(-2)^2\right)$ $\left(3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4\right)$
Evidence for • ⁶ may be implied by • ⁷ , but • ⁷ must be consistent with • ⁵ .		
• ⁷ pd	evaluate the area on one side	• ⁷ $\frac{16}{3}$ $\frac{63}{4}$
• ⁸ ss	interpret integrand with limits of the other area	• ⁸ $\int_0^3 (2x + 4) - (x^3 - x^2 - 4x + 4) dx$ $\int_{-2}^0 (x^3 - x^2 - 4x + 4) - (2x + 4) dx$
• ⁹ pd	evaluate the area on the other side	• ⁹ $\frac{63}{4}$ $\frac{16}{3}$
• ¹⁰ ic	state total area	• ¹⁰ $21\frac{1}{12}$ or $\frac{253}{12}$ or $21 \cdot 1$ $21\frac{1}{12}$ or $\frac{253}{12}$ or $21 \cdot 1$

10

Notes

- The evidence for •² may not appear until •⁸ stage.
- The evidence for •² may appear in a diagram e.g.
- Where a candidate differentiates at •⁵, then •⁵, •⁶, •⁷ and •⁹ are not available.
- Candidates who substitute at •⁶, without integrating at •⁵ lose •⁵, •⁶ and •⁷. However •⁸, •⁹ and •¹⁰ are still available.



Regularly occurring responses

General comment to markers

In this question you should scan the entire response before starting to mark. Where errors occur in the integration/evaluation, use •³ to •⁷ to mark the better solution and •⁸ and •⁹ to mark the poorer solution.

A tabular approach to allocating marks is particularly useful in questions like this, where a candidate's response is spread over several pages, or contains working which appears randomly set out. Response 1 indicates the approach to take here.

Response 1

$$\int_{-2}^3 \text{upper - lower}$$

$$\int_{-2}^3 x^3 - x^2 - 6x dx$$

$$\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$$

$$\left(\frac{3^4}{4} - \frac{3^3}{3} - 3 \cdot 3^2\right) - \left(\frac{-2^4}{4} - \frac{-2^3}{3} - 3 \cdot -2^2\right)$$

$$= \frac{125}{12}$$

- ¹ ✓
- ² ✗
- ³ ✗
- ⁴ ✗
- ⁵ ✓
- ⁶ ✓
- ⁷ ✗
- ⁸ ✗
- ⁹ ✗
- ¹⁰ ✗

Response 2

$$\int_{-2}^0 \dots = \frac{16}{3}$$

$$\int_0^3 6x + x^2 - x^3 dx = \frac{63}{4}$$

$$\int_1^2 x^3 - x^2 - 4x + 4 dx = -\frac{7}{12}$$

$$\text{Total area} = \frac{16}{3} + \frac{63}{4} + \frac{7}{12} = \frac{260}{12}$$

- ¹ ✓
- ² ✓
- ³ ✓
- ⁴ ✓
- ⁵ ✓
- ⁶ ✓
- ⁷ ✓
- ⁸ ✗
- ⁹ ✗
- ¹⁰ ✗

The appearance of this integral is sufficient to lose •⁸ and •⁹.

Response 3

$$\int_1^2 x^3 - x^2 - 4x + 4 dx \quad \checkmark \bullet^1$$

$$\frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x \quad \checkmark \bullet^5$$

$$\left(\frac{2^4}{4} - \frac{2^3}{3} - 2 \cdot 2^2 + 4 \cdot 2\right) - \left(\frac{1^4}{4} - \frac{1^3}{3} - 2 \cdot 1 + 4\right) \quad \checkmark \bullet^6$$

$$= -\frac{7}{12} \quad \text{✗} \bullet^7$$

Candidates who evaluate an integral and obtain a negative answer must deal with this correctly.

The minimum evidence would be

e.g. $\int_{-2}^0 \dots = -\frac{16}{3}$

$$A = \frac{16}{3} \text{ or Area} = \frac{16}{3}$$

N.B. If due to an error the evaluation is negative it must be dealt with correctly. The responses below illustrate what is required under this circumstance. If both integrals lead to negative values only •⁷ or •⁹ is lost.

Response 4A

$$\int_0^3 \dots \frac{63}{4}$$

$$\int_{-2}^0 6x + x^2 - x^3 dx \quad \text{✗} \bullet^8$$

$$= \dots$$

$$= -\frac{16}{3} \quad \text{✗} \bullet^9$$

$$\text{Area} = \frac{63}{4} + -\frac{16}{3} \quad \text{✗} \bullet^{10}$$

$$= \frac{125}{12}$$

Response 4B

$$\int_0^3 \dots \frac{63}{4}$$

$$\int_{-2}^0 6x + x^2 - x^3 dx \quad \text{✗} \bullet^8$$

$$= \dots$$

$$= -\frac{16}{3} \quad \text{✗} \bullet^9$$

$$\text{Area} = \frac{63}{4} + \frac{16}{3} = \frac{253}{12} \quad \checkmark \bullet^{10}$$

Response 4C

$$\int_0^3 \dots \frac{63}{4}$$

$$\int_{-2}^0 6x + x^2 - x^3 dx \quad \text{✗} \bullet^8$$

$$= \dots$$

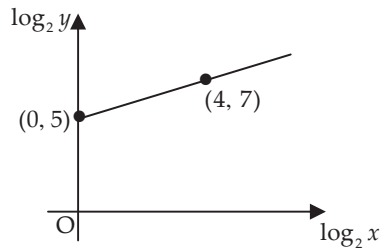
$$= -\frac{16}{3} \text{ can't be negative}$$

$$= \frac{16}{3} \quad \text{✗} \bullet^9$$

$$\text{Area} = \frac{63}{4} + \frac{16}{3} = \frac{253}{12} \quad \checkmark \bullet^{10}$$

5 Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.



Find the values of k and n .

5

Generic Scheme

Illustrative Scheme

- 5
- Method 1
- ¹ ss introduce logarithms to $y = kx^n$
 - ² ic use laws of logarithms
 - ³ ic interpret intercept
 - ⁴ ic solve for k
 - ⁵ ic interpret gradient
- Method 2
- ¹ ss state linear equation
 - ² ic introduce logarithms
 - ³ ic use laws of logarithms
 - ⁴ ic use laws of logarithms
 - ⁵ ic interpret result
- Method 3
- ¹ ic interpret point on log. graph
 - ² ic convert from log. to exp. form
 - ³ ic interpret point and convert
 - ⁴ ss know to substitute points
 - ⁵ ic interpret result

- Method 1
- ¹ $\log_2 y = \log_2 kx^n$ **stated explicitly**
 - ² $\log_2 y = n \log_2 x + \log_2 k$ **stated explicitly**
 - ³ $\log_2 k = 5$ **or** $\log_2 y = 5$ Accept without working
 - ⁴ $k = 32$ **or** 2^5
 - ⁵ $n = \frac{1}{2}$ Accept without working
- Method 2
- ¹ $\log_2 y = \frac{1}{2} \log_2 x + 5$
 - ² $\dots + 5 \log_2 2$ **or** $\dots + \log_2 2^5$
 - ³ $\log_2 y = \log_2 x^{\frac{1}{2}} + \dots$
 - ⁴ $\log_2 y = \log_2 2^5 x^{\frac{1}{2}}$
 - ⁵ $y = 2^5 x^{\frac{1}{2}}$
- ¹ $\log_2 y = \frac{1}{2} \log_2 x + 5$
 - ² $\log_2 y = \log_2 x^{\frac{1}{2}} + 5$
 - ³ $\log_2 \left(\frac{y}{x^{\frac{1}{2}}} \right) = 5$
 - ⁴ $\frac{y}{x^{\frac{1}{2}}} = 2^5$
 - ⁵ $y = 2^5 x^{\frac{1}{2}}$
- Method 3
- ¹ $\log_2 x = 4$ **and** $\log_2 y = 7$
 - ² $x = 2^4$ **and** $y = 2^7$
 - ³ $\begin{cases} \log_2 x = 0 \text{ and } \log_2 y = 5 \\ x = 1 \text{ and } y = 2^5 \end{cases}$
 - ⁴ $2^7 = k \times (2^4)^n$ **and** $2^5 = k$ (from $2^5 = k \cdot 1^n$)
 - ⁵ $n = \frac{1}{2}$

5

Notes

1. Omission of base 2 is treated as bad form at the •¹ and •² stage.
2. Gradient $(m) = \frac{1}{2}$ is not sufficient for •⁵.
3. Throughout this question accept 32 in lieu of 2^5 .
4. Markers should not pick and choose within methods. Use the method which gives the candidate the highest mark.

Regularly occurring responses

Response 1A

With no working

$k = 32 \checkmark \bullet^3$

$n = \frac{1}{2} \checkmark \bullet^5$

2 marks out of 5

Response 1B

With no working

$k = \frac{1}{2} \times \bullet^3$

$n = 32 \times \bullet^5$

0 marks out of 5

Response 2 (Method 1)

$\log_2 k = 5 \checkmark \bullet^3$

$k = 32 \checkmark \bullet^4$

$n = \frac{1}{2} \checkmark \bullet^5$

3 marks out of 5

Response 3 (Variation of Method 2 and Response 1A)

$\log_2 y = \frac{1}{2} \log_2 x + 5 \checkmark \bullet^1$

$\log_2 y = \log_2 \sqrt{x} + 5 \checkmark \bullet^2$

$y = \sqrt{x} + 5$

$k = 1,$

$n = \frac{1}{2} \checkmark \bullet^5$

3 marks out of 5

Response 4 (Variation of Method 2 and Response 1A)

$y = \frac{1}{2}x + 5$

$\log_2 y = \frac{1}{2} \log_2 x + 5 \checkmark \bullet^1$

$\log_2 y - \log_2 x^{\frac{1}{2}} = 5 \checkmark \bullet^2$

$\frac{y}{\sqrt{x}} = 5 \times$

$y = 5\sqrt{x}$

$k = 5 \times$

$n = \frac{1}{2} \checkmark \bullet^5$

3 marks out of 5

- 6 (a) The expression $3 \sin x - 5 \cos x$ can be written in the form $R \sin(x + a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of R and a .

4

Generic Scheme

Illustrative Scheme

6 (a)

- ¹ ss use compound angle formula
- ² ic compare coefficients
- ³ pd process R
- ⁴ pd process a

- ¹ $R \sin x \cos a + R \cos x \sin a$ **stated explicitly**
- ² $R \cos a = 3$ and $R \sin a = -5$ **stated explicitly**
- ³ $\sqrt{34}$ (Accept 5.8) with or without working
- ⁴ 5.253 (Accept 5.3) **must be consistent with** •²

4

Notes

- Treat as bad form the use of k instead of R .
- Treat $R \sin x \cos a + \cos x \sin a$ as bad form only if the equations at the •² stage both contain R .
- $\sqrt{34} \sin x \cos a + \sqrt{34} \cos x \sin a$ or $\sqrt{34}(\sin x \cos a + \cos x \sin a)$ is acceptable for •¹ and •³.
- ² is not available for $R \cos x = 3$ and $R \sin x = -5$, however, •⁴ is still available.
- ⁴ is only available for a single value of a .
- Candidates who work in degrees and don't convert to radian measure lose •⁴. Do not accept $\frac{301\pi}{180}$ or $\frac{5\pi}{3}$.
- Candidates may use any form of the wave equation for •¹, •² and •³, however, •⁴ is only available if the value of a is interpreted for the form $R \sin(x + a)$.

Regularly occurring responses

For •² and •⁴

Response 1A

$$R \cos a = 3 \quad R \sin a = 5 \quad \times \quad \bullet^2$$

$$\tan a = \frac{5}{3}$$

$$a = 1.03 \quad \times \quad \bullet^4$$

Response 1B

$$R \cos a = 3 \quad R \sin a = 5 \quad \times \quad \bullet^2$$

$$\tan a = \frac{3}{5}$$

$$a = 0.54 \quad \times \quad \bullet^4$$

Response 1C

$$R \cos a = 3 \quad R \sin a = -5 \quad \checkmark \quad \bullet^2$$

$$\tan a = -\frac{3}{5}$$

$$a = 5.74 \quad \times \quad \bullet^4$$

Response 2

$$R \sin(x - a) = R \sin x \cos a - R \cos x \sin a \quad \times \quad \bullet^1$$

$$R \cos a = 3 \quad R \sin a = 5 \quad \times \quad \bullet^2$$

$$R = \sqrt{34} \quad \checkmark \quad \bullet^3$$

$$a = 1.03 \quad \times \quad \bullet^4$$

See note 7

3 marks out of 4

Response 3

$$k \sin x \cos a + k \cos x \sin a \quad \checkmark \quad \bullet^1$$

$$\cos a = 3 \quad \sin a = -5 \quad \times \quad \bullet^2$$

$$R = \sqrt{34} \quad \times \quad \bullet^3$$

$$a = 5.3 \quad \times \quad \bullet^4$$

Not consistent with working at •²

2 marks out of 4

6 (b) Hence find the value of t , where $0 \leq t \leq 2$, for which

$$\int_0^t (3 \cos x + 5 \sin x) dx = 3$$

7

Generic Scheme

Illustrative Scheme

6 (b)

- ⁵ pd integrate given expression
- ⁶ ic substitute limits
- ⁷ pd process limits
- ⁸ ss know to use wave equation
- ⁹ ic write in standard format
- ¹⁰ ss start to solve equation
- ¹¹ pd complete and state solution

- ⁵ $3 \sin x - 5 \cos x$
- ⁶ $(3 \sin t - 5 \cos t) - (3 \sin 0 - 5 \cos 0)$
- ⁷ $3 \sin t - 5 \cos t + 5$
- ⁸ $\sqrt{34} \sin(t + 5 \cdot 3) + 5$
- ⁹ $\sin(t + 5 \cdot 3) = -\frac{2}{\sqrt{34}}$
- ¹⁰ $t + 5 \cdot 3 = 3 \cdot 5$ and $5 \cdot 9$
- ¹¹ $t = 0 \cdot 6$

•⁵ to •¹¹ are available to candidates who chose to write this integrand as new wave function.

7

Notes

8. The inclusion of “+ c” at •⁵ stage should be treated as bad form.
9. For those candidates who use a as $5 \cdot 253$ or $5 \cdot 25\dots$, follow through their working for •⁸ to •¹¹.
10. Candidates who use degree measure in (a) lose •⁴ and if they continue to do so in (b), only •⁵, •⁶, •⁷ and •⁸ are available (see also response 6A and 6B below.)

Regularly occurring responses

Response 4 (No integration)

$$\int_0^t 3 \cos x + 5 \sin x dx = \sqrt{34} \sin(x + 5 \cdot 3)$$

lose •⁵, •⁶, •⁷ and •⁸
then

$$\sin(x + 5 \cdot 3) = \frac{3}{\sqrt{34}} \quad \times \quad \bullet^9$$

$$x + 5 \cdot 3 = 0 \cdot 5, 2 \cdot 6, 6 \cdot 8 \quad \times \quad \bullet^{10}$$

$$x = 1 \cdot 5 \quad \times \quad \bullet^{11}$$

Needs to be in terms of t

Response 5

$$\dots 3 \sin x - 5 \cos x \quad \checkmark \quad \bullet^5$$

$$3 \sin t - 5 \cos t - 0 \quad \times \quad \bullet^6 \quad \times \quad \bullet^7$$

$$\sqrt{34} \sin(t + 5 \cdot 3) \quad \checkmark \quad \bullet^8$$

$$\sin(t + 5 \cdot 3) = \frac{3}{\sqrt{34}} \quad \checkmark \quad \bullet^9$$

$$t + 5 \cdot 3 = 0 \cdot 5, 2 \cdot 6, 6 \cdot 8 \quad \checkmark \quad \bullet^{10}$$

$$t = 1 \cdot 5 \quad \checkmark \quad \bullet^{11}$$

Response 6A (Misreading question)

$$\int \sqrt{34} \sin(x + 5 \cdot 3) dx \quad \times \quad \bullet^5 \quad \wedge \quad \bullet^8$$

$$= -\sqrt{34} \cos(t + 5 \cdot 3) = 3 \quad \times \quad \bullet^6 \quad \times \quad \bullet^7$$

$$\cos(t + 5 \cdot 3) = -\frac{3}{\sqrt{34}} \quad \times \quad \bullet^9$$

$$t + 5 \cdot 3 = 2 \cdot 1, 4 \cdot 2, 8 \cdot 4 \quad \times \quad \bullet^{10}$$

$$t = 3 \cdot 1 \text{ i.e. no solution} \quad \times \quad \bullet^{11}$$

3 marks out of 7

If a is left in degrees no marks are available.

Response 6B (Misreading question)

$$\int \sqrt{34} \sin(x + 5 \cdot 3) dx \quad \times \quad \bullet^5 \quad \wedge \quad \bullet^8$$

$$= [-\sqrt{34} \cos(x + 5 \cdot 3)]_0^t$$

$$= -\sqrt{34} \cos(t + 5 \cdot 3) + \sqrt{34} \cos 5 \cdot 3 \quad \checkmark \quad \bullet^6$$

$$-\sqrt{34} \cos(t + 5 \cdot 3) + 3 \cdot 2 \quad \checkmark \quad \bullet^7$$

$$\cos(t + 5 \cdot 3) = \frac{-0 \cdot 2}{-\sqrt{34}} \quad \times \quad \bullet^9$$

$$t + 5 \cdot 3 = 1 \cdot 5, 4 \cdot 7, 7 \cdot 8 \quad \checkmark \quad \bullet^{10}$$

$$t = 2 \cdot 5 \text{ i.e. no solution} \quad \times \quad \bullet^{11}$$

5 marks out of 7

7 Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1 .

The circles have no points of contact.

What is the range of values of p ?

9

Generic Scheme

Illustrative Scheme

7

- ¹ ic state centre of C_1
- ² ic state radius of C_1
- ³ ic state centre of C_2
- ⁴ pd find radius of C_2 in terms of p
- ⁵ ic interpret upper bound for p
- ⁶ ic find distance between centres (d)
- ⁷ ss identify relevant relationship
- ⁸ ic develop relationship by squaring
- ⁹ pd find lower bound for p

- ¹ $(-1, 1)$
- ² 11 Do not accept $\sqrt{121}$
- ³ $(2, -3)$
- ⁴ $\sqrt{13-p}$ Accept c in lieu of p
- ⁵ $p < 13$
- ⁶ 5 **stated explicitly**
- ⁷ $\sqrt{13-p} < 6$ or $r_2 + d < 11$ or $r_2 < 6$
- ⁸ $13 - p < 36$
- ⁹ $p > -23$

9

Notes

1. Treat as bad form the use of c in lieu of p .
2. The evidence for •⁷ must involve an inequality, but may be in words.
3. Treat $\sqrt{13-p}$ as bad form as long as it is clear that the candidate is using $\sqrt{13-p}$.
4. Candidates who are only working with an equation lose both •⁷ and •⁹, however, •⁸ may still be available.
5. •⁹ is only available to candidates who solve an inequation involving a negative coefficient of p .

Regularly occurring responses

Response 1A

Marks 1 to 3 gained

\wedge •⁵ \wedge •⁶
 $\sqrt{-2^2 + 3^2 - p} < 11$
 $\sqrt{13-p} < 11$ ✗ •⁷ ✓ •⁴
 $13-p < 121$ ✗ •⁸
 $p > -108$ ✗ •⁹

Response 1B

$C_1 = (-1, 1)$ ✓ •¹ $C_2 = (2, -3)$ ✓ •³
 $r_1 = 11$ ✓ •² $r_2 = \sqrt{13+p}$ ✗ •⁴
 $d = 5$ ✓ •⁶
 $\sqrt{13+p} < 11$ ✗ •⁷
 $13+p < 121$ ✗ •⁸
 $p < 108$ ✗ •⁹

Response 2

For marks 7 to 9
 $\sqrt{13-p} < 6$ ✓ •⁷
 $\sqrt{13-p} < 6$ ✗ •⁸
 $\underline{169-p} < 36$
 $-p < -133$
 $p > 133$ ✗ •⁹

Penalise the use of \leq and/or \geq once only.

Response 3 (see note 4)

$\sqrt{13-p} = 0$
 $p = 13$ ✗ •⁵
 $\sqrt{13-p} = 6$ ✗ •⁷
 $13-p = 36$ ✗ •⁸
 $p > -23$ ✗ •⁹

Response 4

$\sqrt{13-p} \geq 0$
 $p \leq 13$ ✗ •⁵
 $\sqrt{13-p} \leq 6$ ✗ •⁷
 $p \geq -23$ ✗ •⁹

Response 5

$0 < \sqrt{13-p} < 6$ ✓ •⁷
 $0 < 13-p < 36$ ✓ •⁸
 $-13 < -p < 23$
 \checkmark •⁵
so $p < 13$ and $p > -23$ ✓ •⁹
 \checkmark •⁹
or $-23 < p < 13$ ✓ •⁵

Regularly occurring responses

Response 6

$$(x-2)^2 + (y+3)^2 = 13-p \quad \times$$

$$13-p < 121 \quad \times \bullet^4 \quad \times \bullet^7$$

$$p > -108 \quad \times \bullet^9$$