X100/12/03

NATIONAL QUALIFICATIONS 2012

MONDAY, 21 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between **a** and **b**

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted. *Marks*

1. Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

(*b*) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots. **3**

2. (*a*) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

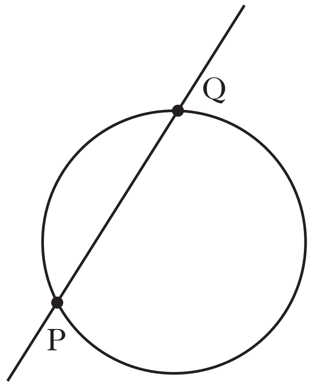


Diagram 1

Find the coordinates of P and Q. **6**

(*b*) Diagram 2 shows the circle from (*a*) and a second congruent circle, which also passes through P and Q.

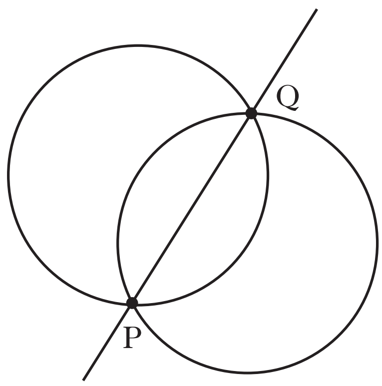


Diagram 2

6

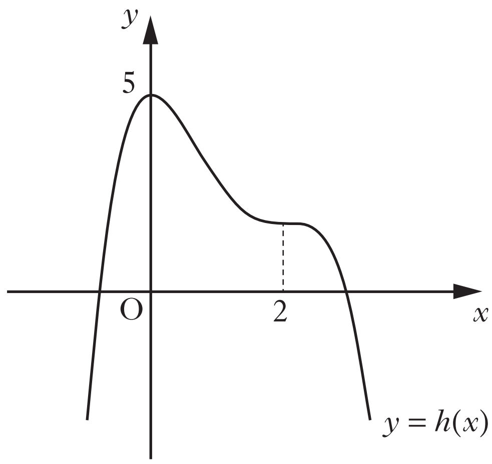
Determine the equation of this second circle.

Marks

3. A function f is defined on the domain $0 \le x \le 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.

Determine the maximum and minimum values of f. **7**

4. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(*a*) $y = h'(x)$; **3**

(*b*) $y = 2 - h'(x)$. **3**

5. A is the point $(3, -3, 0)$, B is $(2, -3, 1)$ and C is $(4, k, 0)$.

(*a*) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

(ii) Show that $\cos A\hat{B}C = \dfrac{3}{\sqrt{2(k^2 + 6k + 14)}}$. **7**

(*b*) If angle ABC = 30°, find the possible values of k. **5**

Marks

6. For $0 < x < \frac{\pi}{2}$, sequences can be generated using the recurrence relation

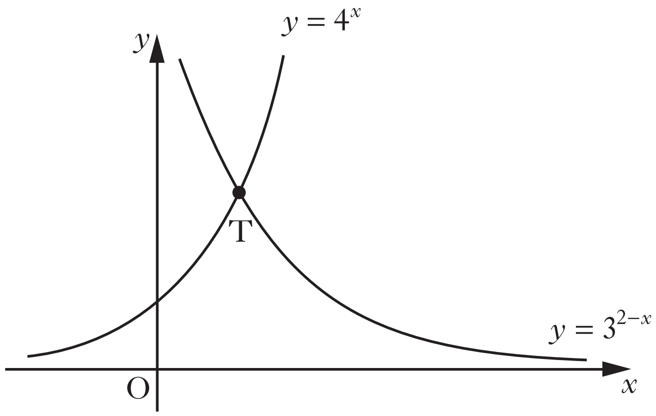
$$u_{n+1} = (\sin x)u_n + \cos 2x, \text{ with } u_0 = 1.$$

(*a*) Why do these sequences have a limit? **2**

(*b*) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$.

Find the value(s) of x. **7**

7. The diagram shows the curves with equations $y = 4^x$ and $y = 3^{2-x}$.



The graphs intersect at the point T.

(*a*) Show that the x – coordinate of T can be written in the form $\dfrac{\log_a p}{\log_a q}$,

for all $a > 1$. **6**

(*b*) Calculate the y – coordinate of T. **2**

[*END OF QUESTION PAPER*]

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