X100/12/02

NATIONAL QUALIFICATIONS 2013

WEDNESDAY, 22 MAY
1.00 PM – 2.30 PM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

LI X100/12/02 6/30310

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

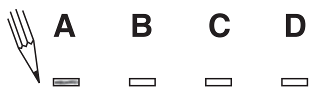
A 8

B 1

C 0

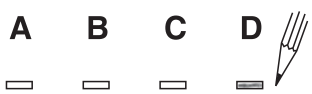
D -4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and, using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

1. The functions f and g are defined by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$, on the set of real numbers.

Find $g(f(x))$.

A $3x^2 - 1$

B $9x^2 - 15$

C $9x^2 + 17$

D $3x^3 - 4x^2 + 3x - 4$

2. The point P (5, 12) lies on the curve with equation $y = x^2 - 4x + 7$.

What is the gradient of the tangent to this curve at P?

A 2

B 6

C 12

D 13

3. Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.

A -32

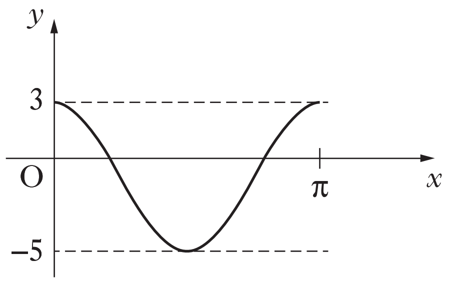
B -24

C 48

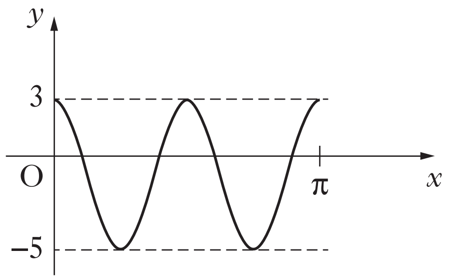
D 56

4. Which of the following shows the graph of $y = 4\cos 2x - 1$, for $0 \leq x \leq \pi$?

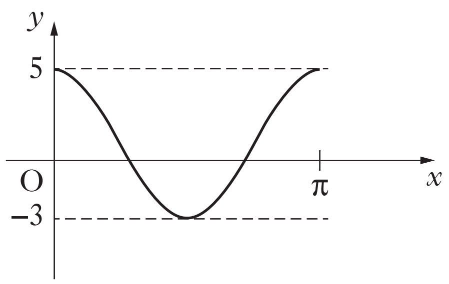
A



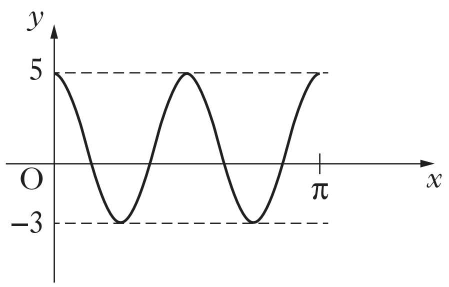
B



C



D



[Turn over

5. The line L passes through the point $(-2, -1)$ and is parallel to the line with equation $5x + 3y - 6 = 0$.

What is the equation of L?

A $3x + 5y - 11 = 0$

B $3x + 5y + 11 = 0$

C $5x + 3y - 13 = 0$

D $5x + 3y + 13 = 0$

6. What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x - 2)$?

A 0

B 3

C 4

D 8

7. Find $\int x(3x + 2)\, dx$.

A $x^3 + c$

B $x^3 + x^2 + c$

C $\frac{1}{2}x^2\left(\frac{3}{2}x^2 + 2x\right) + c$

D $3x^2 + 2x + c$

8. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}1u_n + 8$, with $u_1 = 11$.

Here are two statements about this sequence:

(1) $u_0 = 9{\cdot}1$;

(2) The sequence has a limit as $n \longrightarrow \infty$.

Which of the following is true?

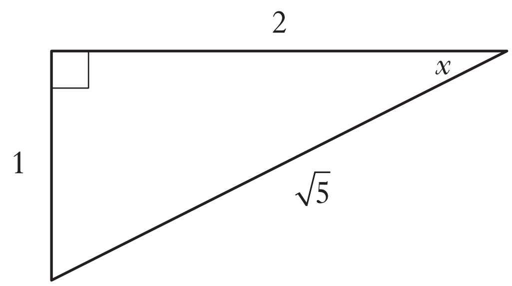
A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

9. The diagram shows a right-angled triangle with sides and angles as marked.



Find the value of $\sin 2x$.

A $\frac{4}{5}$

B $\frac{2}{5}$

C $\frac{2}{\sqrt{5}}$

D $\frac{1}{\sqrt{5}}$

10. If $0 < a < 90$, which of the following is equivalent to $\cos(270 - a)°$?

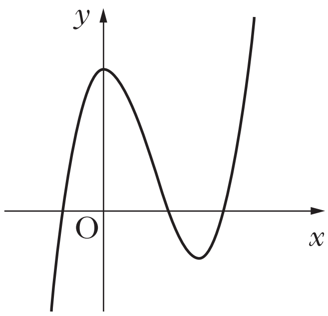
A $\cos a°$

B $\sin a°$

C $-\cos a°$

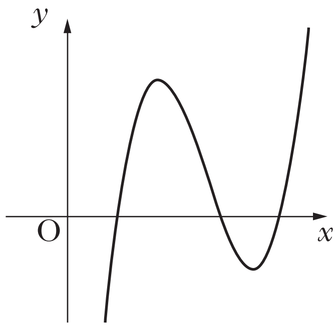
D $-\sin a°$

11. The diagram shows a cubic curve with equation $y = f(x)$.

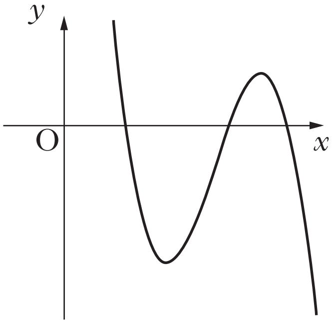


Which of the following diagrams could show the curve with equation $y = -f(x - k)$, $k > 0$?

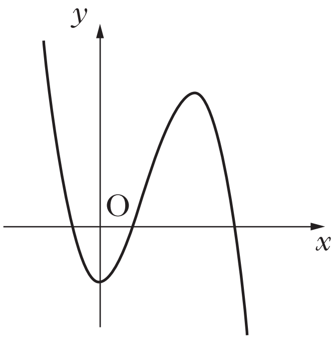
A



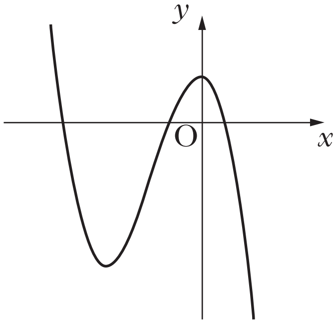
B



C



D



12. If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, find $|\mathbf{f} + \mathbf{g}|$.

A $\sqrt{14}$ units

B $\sqrt{42}$ units

C $\sqrt{66}$ units

D $\sqrt{70}$ units

13. A function f is defined on a suitable domain by $f(x) = \dfrac{x+2}{x^2 - 7x + 12}$.

What value(s) of x cannot be in this domain?

A 3 and 4

B -3 and -4

C -2

D 0

14. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\mathbf{a}.\mathbf{b} = 5$, what is the value of $\mathbf{a}.(\mathbf{a} + \mathbf{b})$?

A 11

B 14

C 15

D 21

15. Solve $\tan\left(\dfrac{x}{2}\right) = -1$ for $0 \leq x < 2\pi$.

A $\dfrac{\pi}{2}$

B $\dfrac{7\pi}{8}$

C $\dfrac{3\pi}{2}$

D $\dfrac{15\pi}{8}$

[Turn over

16. Find $\int (1-6x)^{-\frac{1}{2}} dx$ where $x < \frac{1}{6}$.

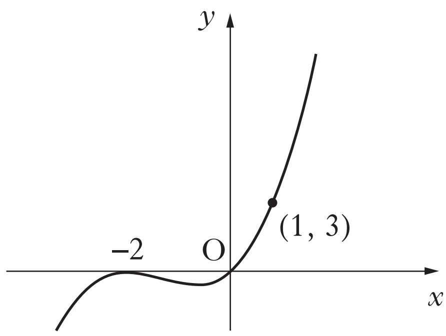
A $\quad \frac{1}{9}(1-6x)^{-\frac{3}{2}} + c$

B $\quad 3(1-6x)^{-\frac{3}{2}} + c$

C $\quad -\frac{1}{3}(1-6x)^{\frac{1}{2}} + c$

D $\quad -3(1-6x)^{\frac{1}{2}} + c$

17. The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points $(-2, 0)$, $(0, 0)$ and $(1, 3)$.



What are the values of a and k?

	a	k
A	-2	$\frac{1}{3}$
B	-2	3
C	2	$\frac{1}{3}$
D	2	3

18. Given that $y = \sin(x^2 - 3)$, find $\frac{dy}{dx}$.

A $\sin 2x$

B $\cos 2x$

C $2x \sin(x^2 - 3)$

D $2x \cos(x^2 - 3)$

19. Solve $1 - 2x - 3x^2 > 0$, where x is a real number.

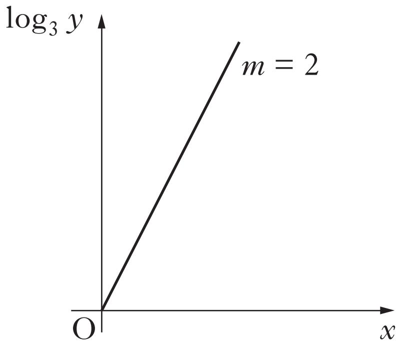
A $x < -1$ or $x > \frac{1}{3}$

B $-1 < x < \frac{1}{3}$

C $x < -\frac{1}{3}$ or $x > 1$

D $-\frac{1}{3} < x < 1$

20. The graph of $\log_3 y$ plotted against x is a line through the origin with gradient 2, as shown.



Express y in terms of x.

A $y = 2x$

B $y = 9x$

C $y = 6^x$

D $y = 9^x$

[*END OF SECTION A*]

[Turn over for SECTION B on *Page twelve*

SECTION B

Marks

ALL questions should be attempted.

21. Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. **3**

22. A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.

(*a*) Write down the centre and calculate the radius of C_1. **2**

(*b*) The point P(3, 2) lies on the circle C_1.

Find the equation of the tangent at P. **3**

(*c*) A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1.

Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. **3**

(*d*) Show that the tangent found in part (*b*) is also a tangent to circle C_2. **4**

23. (*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k\sin(x - a)°$, where $k > 0$ and $0 \leq a < 360$.

Calculate the values of k and a. **4**

(*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \leq x < 360$. **2**

24. (*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.

(ii) Find the ratio in which T divides AB. **4**

(*b*) The point C lies on the x-axis.

If TB and TC are perpendicular, find the coordinates of C. **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

X100/12/03

NATIONAL QUALIFICATIONS 2013

WEDNESDAY, 22 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

Read carefully

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LI X100/12/03 6/30310

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ALL questions should be attempted. *Marks*

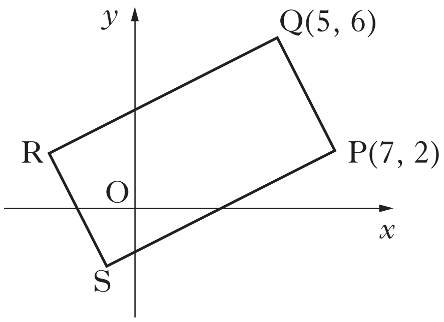
1. The first three terms of a sequence are 4, 7 and 16.

The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

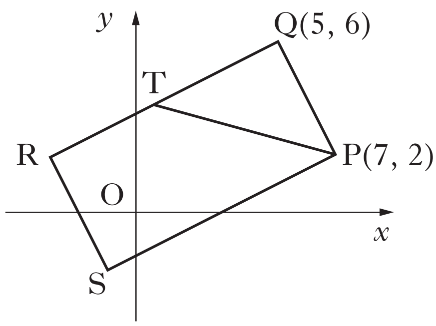
Find the values of m and c. **4**

2. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



(*a*) Find the equation of QR. **3**

(*b*) The line from P with the equation $x + 3y = 13$ intersects QR at T.



Find the coordinates of T. **3**

(*c*) Given that T is the midpoint of QR, find the coordinates of R and S. **3**

[Turn over

Marks

3. (*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. **4**

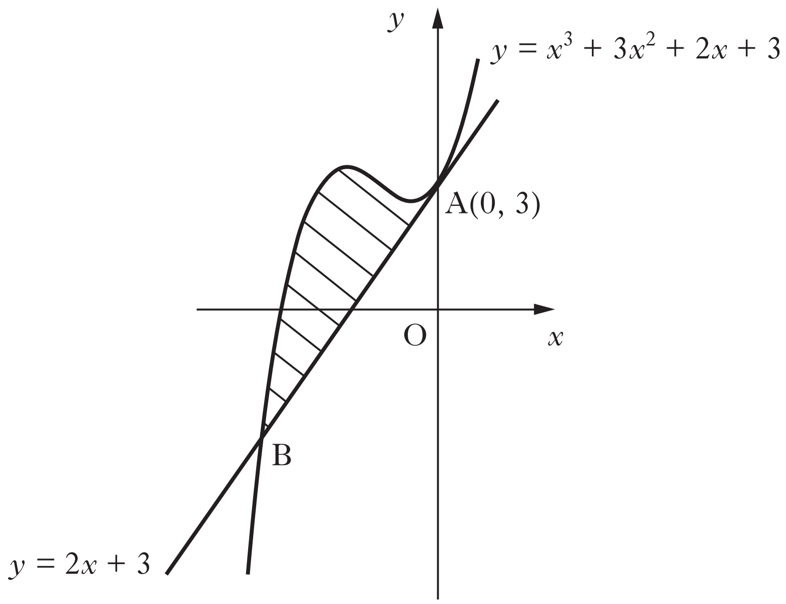
(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point. **5**

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.



The line meets the curve again at B.

Show that B is the point $(-3, -3)$ and find the area enclosed by the line and the curve. **6**

5. Solve the equation

$$\log_5(3 - 2x) + \log_5(2 + x) = 1, \text{ where } x \text{ is a real number.}$$ **4**

Marks

6. Given that $\int_0^a 5\sin 3x \, dx = \frac{10}{3}, \quad 0 \leq a < \pi,$

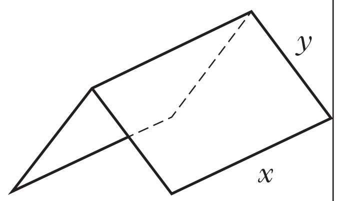
calculate the value of a. **5**

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,\text{m}^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$ **3**

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame. **7**

8. Solve algebraically the equation

$$\sin 2x = 2\cos^2 x \qquad \text{for } 0 \leq x < 2\pi$$ **6**

[Turn over for Question 9 on *Page six*

Marks

9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. **4**

(*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*? **3**

[*END OF QUESTION PAPER*]

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