DINGWALL ACADEMY

NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2010/2011
Paper 1
Assessing Units 1 & 2

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2010/2011 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

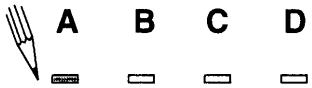
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k, 7)$ lies on this line, the value of k is

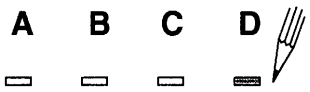
A 2
B 27
C 1·5
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

SECTION A
ALL questions should be attempted

1. A sequence is defined by the recurrence relation

$$U_{n+1} = 3U_n - 6 \text{ with } U_0 = 2 \cdot 4.$$

The value of U_2 is

A $-2 \cdot 4$

B -3

C 0

D 2

2. Two functions are defined on suitable domains as

$$f(x) = 2x - 4 \text{ and } g(x) = x^2 + 1.$$

$f(g(1))$ is

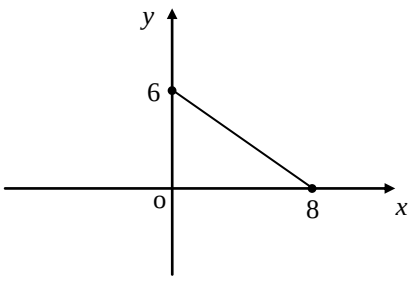
A 3

B -3

C 5

D 0

3.



The line above has as its equation

A $y = -\frac{4}{3}x + 6$

B $y = -\frac{3}{4}x + 6$

C $y = \frac{3}{4}x + 6$

D $y = -\frac{3}{4}x + 8$

4. Given that C is a constant of integration, $\int x^{\frac{1}{4}}\,dx$ equals

A $\frac{5}{4}x^{\frac{5}{4}} + C$

B $\frac{1}{4}x^{-\frac{3}{4}} + C$

C $\frac{4}{5}x^{\frac{5}{4}} + C$

D $\frac{1}{4}x^{\frac{5}{4}} + C$

5. $x^2 - 4x + 2$ can be expressed in the form $(x-a)^2 + b$.

What is the value of b?

A 6

B -4

C -2

D 2

6. What is the value of $\cos\frac{11\pi}{6} + \sin\frac{7\pi}{6}$

A $\frac{\sqrt{3}-1}{2}$

B $-\frac{\sqrt{3}-1}{2}$

C $\frac{\sqrt{3}+1}{2}$

D $-\frac{\sqrt{3}+1}{2}$

7. The function $f(x) = px^2 - 20x$ is such that $f(-2) = 0$.

The value of p is

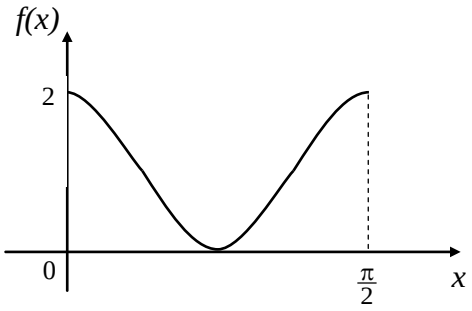
A 10

B 0

C -20

D -10

8. The diagram opposite shows part of the graph of a trigonometrical function.



The most likely function could be $f(x) = ...$

A $2\cos 4x$

B $\cos 4x$

C $\cos 2x + 1$

D $\cos 4x + 1$

9. If $x - 2$ is a factor of the polynomial $x^3 - 2x^2 + kx - 10$, then k equals

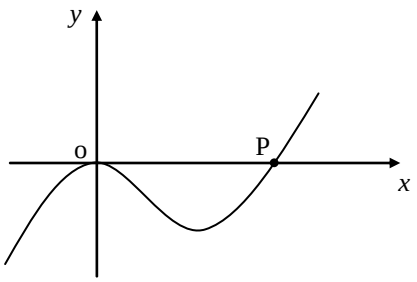
A 5

B -13

C 3

D -5

10. The sketch below shows part of the curve $y = x^3 - 5x^2$.



The gradient of the tangent to the curve at the point P(5, 0) is

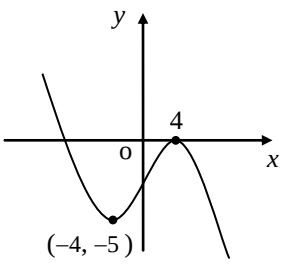
A -125

B 0

C 25

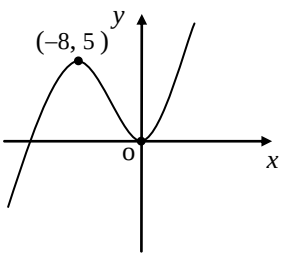
D 50

11. Part of the graph of the function $y = f(x)$ is shown below.

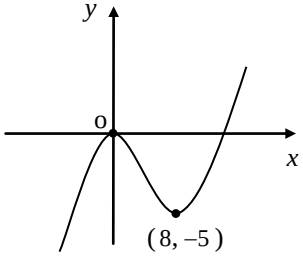


Which of the following graphs represents the related function $y = -f(x+4)$?

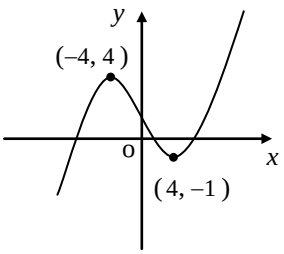
A



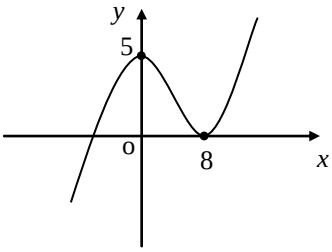
B



C



D



12. The equation of the line passing through $(2, -1)$ and parallel to the line with equation $2x + 3y + 5 = 0$ is

A $y + 1 = \frac{3}{2}(x - 2)$

B $y + 1 = -\frac{2}{3}(x - 2)$

C $y - 2 = -\frac{2}{3}(x + 1)$

D $y + 1 = -2(x - 2)$

13. Which of the following expressions is/are equal to $(9a)^{-\frac{3}{2}}$?

(1) $\dfrac{9}{a^{\frac{3}{2}}}$

(2) $\dfrac{1}{27\sqrt{a^3}}$

(3) $\dfrac{1}{9a^{\frac{3}{2}}}$

A all three expressions

B only expression (1)

C only expression (2)

D expressions (2) and (3)

14. The equation of a circle, centre $(-2, 6)$, with the x-axis as a tangent is

A $(x-6)^2+(y+2)^2=36$

B $(x+2)^2+(y-6)^2=4$

C $(x-2)^2+(y+6)^2=36$

D $(x+2)^2+(y-6)^2=36$

15. The equation $4x^2+k=0$ has real roots. The range of values of k is

A $k \le 0$

B $k > 0$

C $-2 < k < 2$

D $k < 0$

16. A function is defined as $f(x) = x^3 + 1$.

Which of the following statements **is true** about this function?

A it is never increasing

B it is never stationary

C it is never decreasing

D it has two stationary points

17. If $x = \sqrt{3} - 1$, then x^2 equals

A 2

B 4

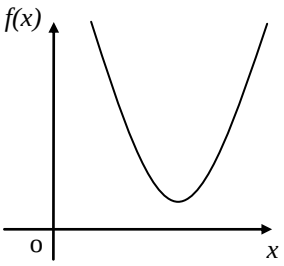
C $10 - 2\sqrt{3}$

D $4 - 2\sqrt{3}$

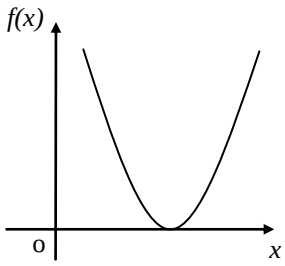
18. A quadratic function, *f*, where $f(x) = ax^2 + bx + c$, is such that $a < 0$ and $b^2 - 4ac < 0$.

Which of the following is a possible sketch of the graph of *f(x)*?

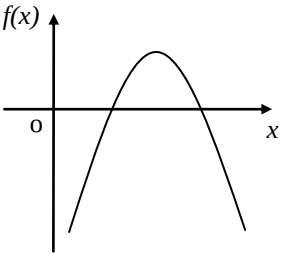
A



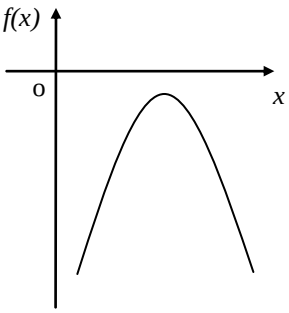
B



C



D



19. If $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, then $\tan\theta$ is equal to

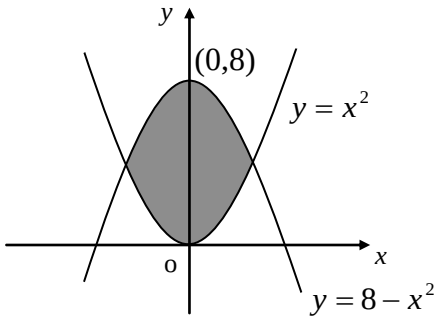
A $-\frac{4}{3}$

B $-\frac{3}{4}$

C $\frac{4}{3}$

D $\frac{3}{4}$

20. The area enclosed between the curves $y = x^2$ and $y = 8 - x^2$ is shown as the shaded area in the diagram below.



Which of the following gives the area of the shaded section?

A $\int_{-\sqrt{8}}^{\sqrt{8}} \left(8 - 2x^2\right) dx$

B $\int_{-2}^{2} \left(8 - 2x^2\right) dx$

C $\int_{-4}^{4} \left(8 - 2x^2\right) dx$

D $\int_{-2}^{2} \left(2x^2 - 8\right) dx$

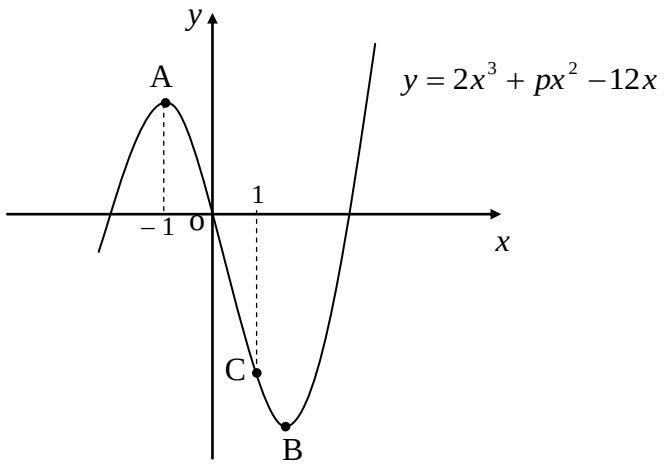
[END OF SECTION A]

SECTION B

ALL questions should be attempted

21. The diagram below, which is not drawn to scale, shows part of the curve with equation $y = 2x^3 + px^2 - 12x$, where p is a constant.

A is a stationary point and has -1 as its x-coordinate.



(a) By considering the derivative of y, and using the x-coordinate of point A to help you, find the value of p. **4**

(b) Establish the coordinates of B the other stationary point.
(all relevant working must be shown) **4**

(c) The point C on the curve has 1 as its x-coordinate.

Find the equation of the tangent to the curve at C. **3**

22. Solve the equation $2\cos^2\theta = \cos\theta + 1$, for $0 < \theta < 4\pi$. **6**

23. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n + 20$, where a is a constant.

(a) Given that $U_3 = 36$ and $U_4 = 38$, find **algebraically**, the value of a. **2**

(b) Hence find the limit of this sequence. **2**

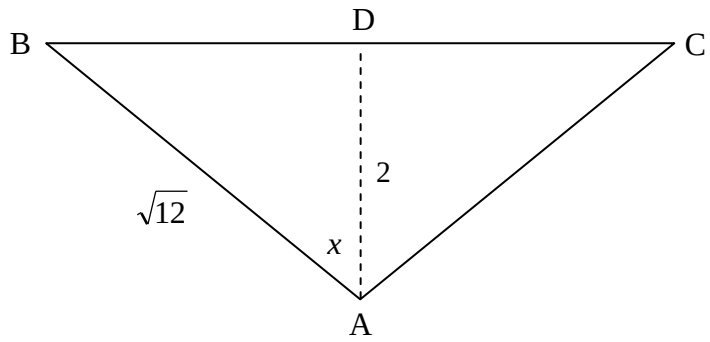
(c) The sequence is such that $L = \boldsymbol{k}U_0$, where L is the limit of the sequence, U_0 is the the initial value and $\boldsymbol{k}$ is a number.

Find the value of $\boldsymbol{k}$. **3**

24. In the diagram below, which is not drawn to scale, triangle ABC is isosceles with AB = AC.

D is the mid-point of BC. AB = $\sqrt{12}$ units and AD = 2 units as shown.

Angle BAD = x.



(a) Show clearly that $\sin x = \frac{\sqrt{2}}{\sqrt{3}}$. **3**

(b) Hence show that $\cos \text{BAC} = -\frac{1}{3}$ **3**

[END OF SECTION B]

[END OF QUESTION PAPER]