NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2010/2011
Paper 2
Assessing Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

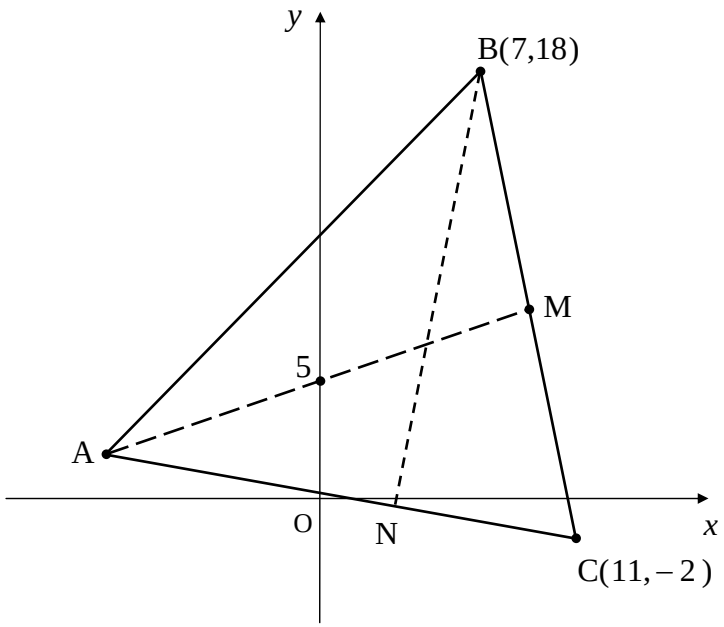
$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

ALL questions should be attempted

1. In the diagram below triangle ABC has two of its vertices as B(7,18) and C(11, – 2).

M is the mid-point of BC. The line AM crosses the *y*-axis at (0, 5).

BN is an altitude of the triangle.



(a) Find the equation of the median AM. **3**

(b) Given that the equation of side AB is $y = x + 11$, establish the coordinates of vertex A. **3**

(c) Hence find the equation of the altitude BN. **3**

2. (a) If $x - 1$ is a factor of $3x^3 + kx^2 + 4x - 13$, find the value of *k*. **3**

(b) Hence find the *x*-coordinate of the single stationary point on the curve with equation $y = 3x^3 + kx^2 + 4x - 13$ when *k* takes this value. **4**

3. Two functions, defined on suitable domains, are given as

$f(x) = 3px + \frac{1}{2}p$ and $g(x) = x(3px - 2)$, where p is a constant.

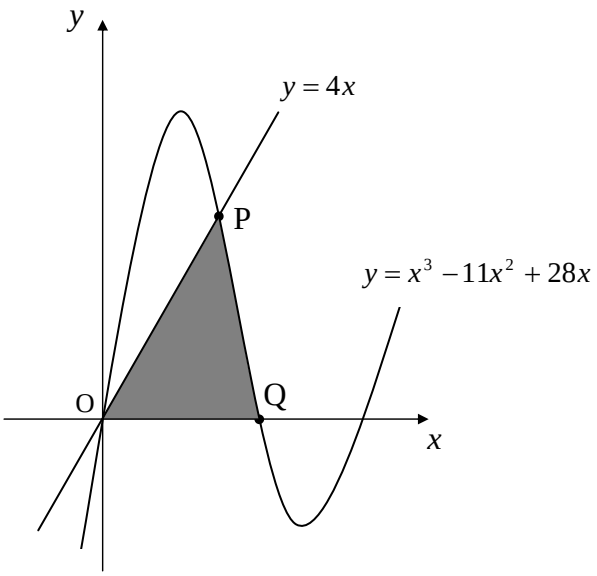
(a) Show clearly that the composite function $f(g(x))$ can be written in the form $f(g(x)) = ax^2 + bx + c$, and write down the values of a, b and c in terms of p. **4**

(b) Hence find the value of p, where $p > 0$, such that the equation $f(g(x)) = 0$ has **equal roots**. **3**

4. The diagram below, **which is not drawn to scale**, shows part of the curve with equation $y = x^3 - 11x^2 + 28x$ and the line $y = 4x$.

The line and the curve intersect at the origin and the point P.

The curve also crosses the x-axis at Q.

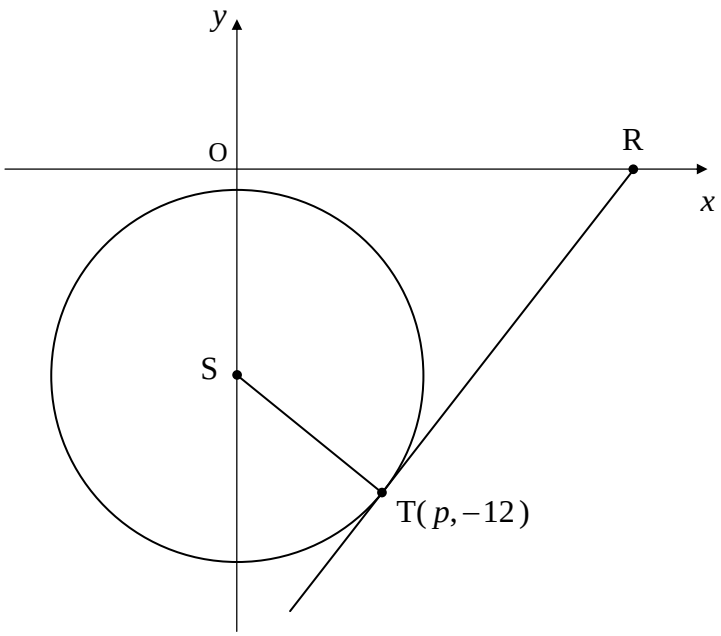


(a) Find the coordinates of P and Q. **5**

(b) Calculate the shaded area in the diagram. **6**

5. The circle, centre S, has as its equation $x^2 + y^2 + 16y + 12 = 0$.

T$(p, -12)$ is a point of tangency.



(a) Find the value of p, the x-coordinate of T. **2**

(b) Write down the coordinates of S, the centre of the circle. **1**

(c) Find the equation of the tangent through T and hence state the coordinates of R. **4**

(d) Establish the equation of the circle which passes through the points S, T and R. **3**

6. The cost of laying one mile of service piping to a wind farm is estimated by means of the formula

$$C = \frac{16200}{9a} + 450a,$$

where C is the cost in tens of pounds and a is the cross-sectional area of the tube in square inches.

What cross-sectional area is the most economical to use? **5**

7. A formula is given as $E = \sin^2\theta - \frac{1}{2}\sin\theta - 1$ for $0 \le \theta \le \frac{\pi}{2}$.

(a) Express E in the form $E = (\sin\theta + p)^2 + q$ and write down the values of p and q. **2**

(b) Hence, or otherwise, state the minimum value of E and the corresponding replacement for θ. Give your answer correct to 2 decimal places. **3**

8. A function, defined on a suitable domain, has as its derivative $f'(x) = 3x^2 - \frac{10}{x^2}$.

(a) Given that $f(2) = 3$, find $f(x)$. **5**

(b) Hence find $f(1)$. **1**

[END OF QUESTION PAPER]