DINGWALL ACADEMY

NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2009/2010
Paper 2
Assessing Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

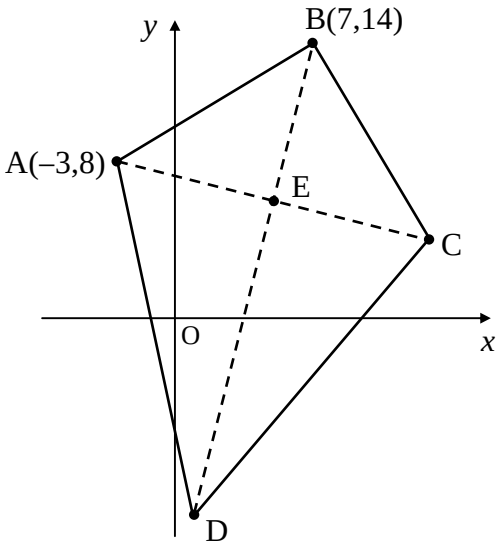
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

ALL questions should be attempted

1. Kite ABCD has two of its vertices at A(–3,8) and B(7,14) as shown.



(a) Given that the equation of the longer diagonal BD is $y = 4x - 14$, find the equation of the short diagonal AC expressing your answer in the form $ax + by + c = 0$ and write down the values of a, b and c. **4**

(b) Find the coordinates of E, the point of intersection of the two diagonals. **3**

(c) Hence establish the coordinates of C. **2**

2. Two functions are defined on suitable domains as $f(x) = 4x + 1$ and $g(x) = \dfrac{1}{x-1}$.

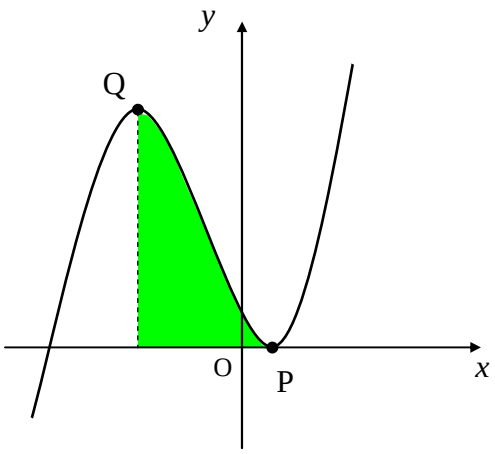
(a) If $h(x) = f(g(x))$, show clearly that $h(x)$ can be written as

$$h(x) = \frac{x+3}{x-1}.$$ **3**

(b) Show that value of $h(\sqrt{5})$ can be expressed in the form $p + \sqrt{5}$ and write down the value of p. **4**

3. A function is defined on the set of real numbers as $f(x) = 2x^3 + 3x^2 - 12x + 7$.

Part of the graph of $y = f(x)$ is shown below.



(a) Find the coordinates of the stationary points P and Q. **5**

(b) Calculate the shaded area in the diagram. **4**

4. Solve algebraically the equation

$$2\sin^2 x = 1 - \sin x \qquad \text{for} \qquad 0 \leq x \leq 360 .$$ **5**

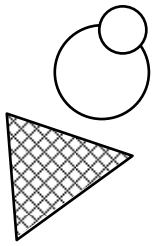
5. A recurrence relationship is such that $U_{n+1} = \frac{a}{4}U_n + 12$.

(a) If $U_0 = 16$ show clearly that $U_2 = a^2 + 3a + 12$. **2**

(b) Hence find a if $U_2 = 30$ and $a > 0$. **3**

(c) Explain why this sequence has a limit and find the limit. **3**

6. An ice-cream manufacturer has decided on a new logo for her company.

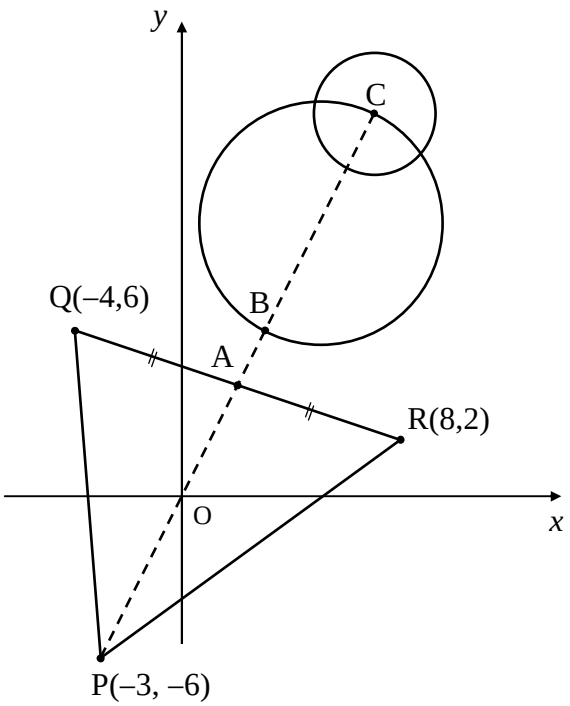


It consists of a triangle and two circles representing
a wafer cone and two balls of ice cream.

Placed on a set of rectangular axes the logo is modelled in the diagram below.

The triangle has coordinates P(–3, –6), Q(–4,6) and R(8,2).

A is the midpoint of QR.



(a) Find the equation of PA **3**

(b) When PA is extended it intersects with the larger circle at B and C.

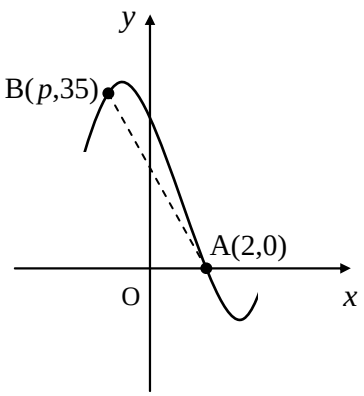
If the larger circle has as its equation $x^2 + y^2 - 10x - 20y + 105 = 0$,
find the coordinates of C. **4**

(c) Given that C is the centre of the smaller circle and that its radius is
exactly half of the larger circle, find the equation of the smaller circle. **3**

7. A curve has as its equation $y = x^3 - kx^2 - 16x + 32$.

Part of the graph of this curve is shown below.

The diagram is not drawn to scale.



(a) If the curve crosses the x-axis at A(2,0), find k. **3**

(b) The point B(p,35) also lies on this curve, find the value of p. **3**

(c) Calculate the size of the angle between the line AB and the x-axis in the positive direction. **Give your answer to the nearest degree**. **2**

8. Given that $\int_0^a (2x)\,dx$ is equal to the **derivative** of $3a^2 - 9a$, find a. **4**

[**END OF QUESTION PAPER**]