DINGWALL ACADEMY

NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2008/2009
Assessing Units 1 & 2
Paper 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

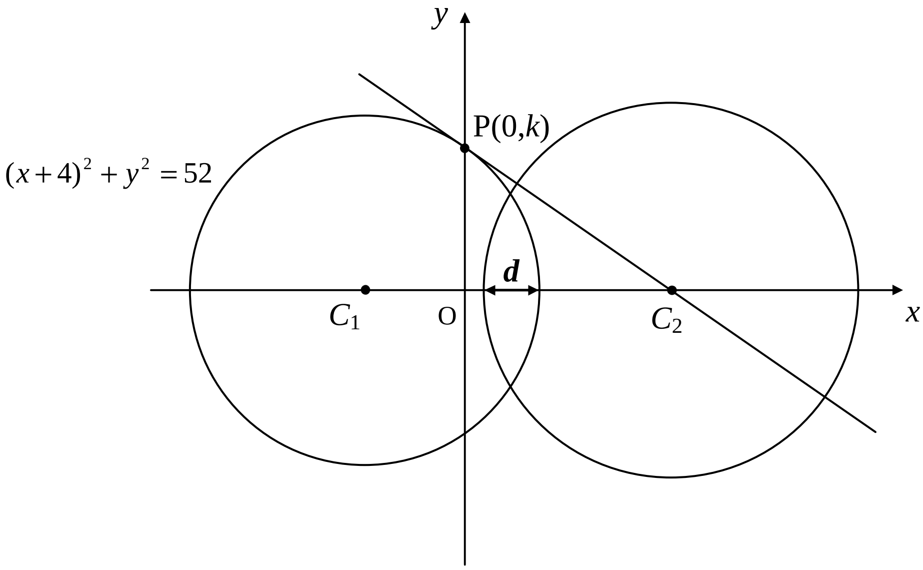
$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

ALL questions should be attempted

1. Consider the diagram below.

The circle centre C_1 has as its equation $(x+4)^2 + y^2 = 52$.
The point P(0, k) lies on the circumference of this circle and the tangent to this circle through P has been drawn.

A second circle with centre C_2 is also shown.



(a) What is the value of k? **2**

(b) Hence find the equation of the tangent through P. **4**

(c) The tangent through P passes through C_2 the centre of the second circle. State the coordinates of C_2. **1**

(d) Given that the second circle has a radius of 8 units, calculate the distance marked ***d*** on the diagram, giving your answer correct to 1 decimal place. **3**

2. Given that f(x) = cos x, and g(x) = $4x^2$ - 1,

Solve algebraically the equation

h(x) = 0 , for $0 \le x \le 360$, where h(x) = g(f(x)).

5

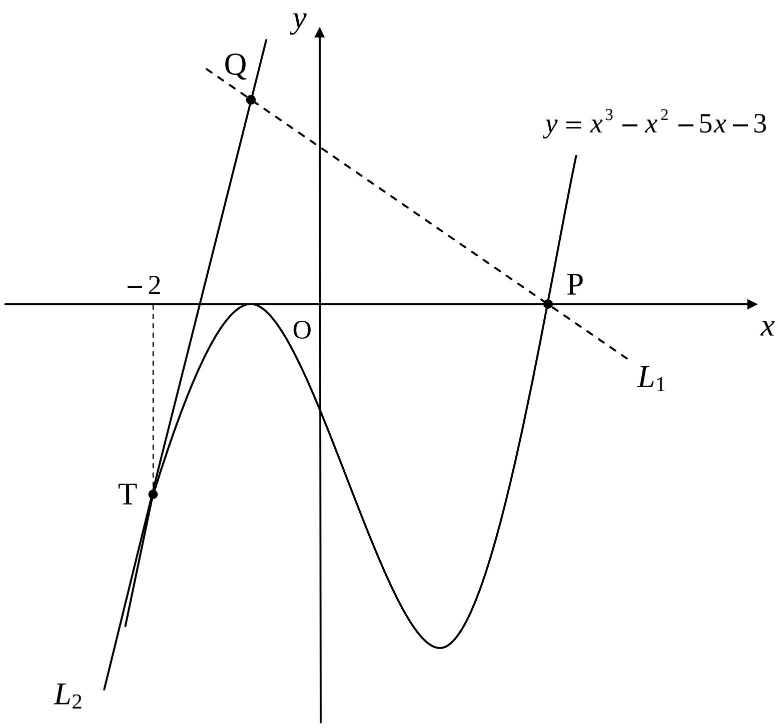
3. A curve has as its derivative $\dfrac{dy}{dx} = 2x - \dfrac{6}{x^2}$.

(a) Given that the point (2, 3) lies on this curve, express y in terms of x. **4**

(b) Hence find p if the point (3, p) also lies on this curve. **1**

4. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve with equation $y = x^3 - x^2 - 5x - 3$.

Two straight lines are also shown, L_1 and L_2.



(a) Find the coordinates of P. **2**

(b) Line L_1 has a gradient of $-\frac{3}{2}$ and passes through the point P.
Find the equation of L_1. **1**

(c) Line L_2 is a tangent to the curve at the point T where $x = -2$.
Find the equation of L_2. **4**

(d) Hence find the coordinates of Q, the point of intersection of the two lines. **3**

5. A company making commercial "glow sticks" have devised a method to test the brightness and consistency of the glow given off.

All glow sticks depend on a chemical process known as chemiluminesence to produce their light. Once a glow stick has been illuminated (by mixing two chemicals together) the brightness of the glow diminishes over a period of time.

When one of their glow sticks is ignited the initial brightness is rated at 200 *gu* (glow units).

(a) During any 1 hour period the glow light is known to lose 8% of its brightness at the beginning of the period.

Calculate the brightness remaining, in *gu*'s, after a period of 4 hours. **2**

(b) At the end of each 4 hour period, the glow light is automatically passed through a refrigerated tube allowing the light to regain some of its lost brightness.
Each pass through this refrigerated tube every four hours allows the glow stick to regain 32 glow units.

The 4 hour cycle described above is now left to run uninterrupted for a total of 16 hours.

By considering an appropriate recurrence relation, calculate the brightness remaining, in *gu*'s, after this 16 hour period has been completed.
Your answer must be accompanied with the appropriate working. **3**

(c) If this cycle was left to run over a very long period of time would the brightness of the glow stick ever drop to below half of its initial brightness?
Explain your answer.
Your answer and explanation must be accompanied with the appropriate working. **3**

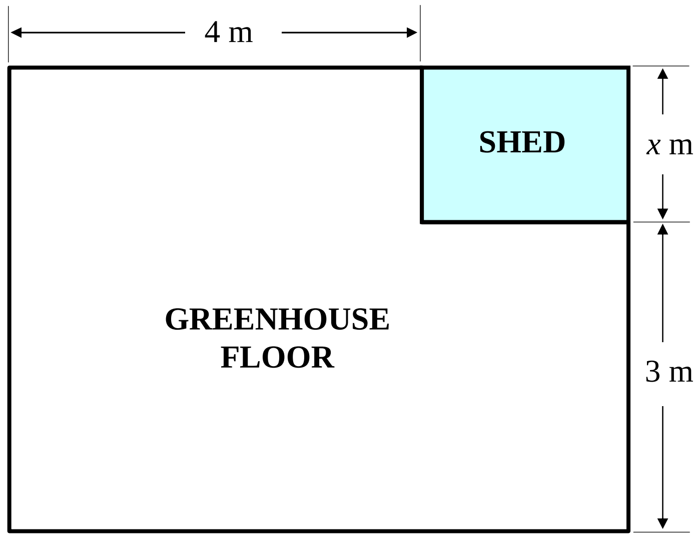
6. (a) If $k = \dfrac{(x-1)^2}{x^2+4}$, where k is a real number, show clearly that

$$(k-1)x^2 + 2x + (4k-1) = 0.$$ **3**

(b) Hence find the value of k given that the equation $(k-1)x^2 + 2x + (4k-1) = 0$ has equal roots and $k > 0$. **5**

7. The floor plan of a rectangular greenhouse is shown below. All dimensions are in metres.

The gardener places a rectangular wooden storage shed, of width x metres, in one corner.



(a) Given that the **area of the shed** is 3 square metres, show clearly that the area of greenhouse floor remaining, A square metres, is given in terms of x as

$$A(x) = 12 + 4x + \frac{9}{x}.$$ **3**

(b) Hence find the value of x which **minimises** the area of the greenhouse floor remaining, **justifying your answer**. **5**

8. Solve the equation,

$\text{Sin}^3\theta - \text{Sin}^2\theta + 4\text{Sin}\theta - 4 = 0$, for $0 \leq \theta \leq 2\pi$ **6**

[**END OF QUESTION PAPER**]