DINGWALL ACADEMY

NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2009/2010
Paper 1
Assessing Units 1 & 2

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2009/2010 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

Sample Question

A line has equation $y = 4x - 1$.

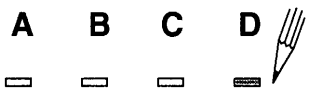
If the point $(k,7)$ lies on this line, the value of k is

A 2
B 27
C 1·5
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).

Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

SECTION A
ALL questions should be attempted

1. A straight line with gradient 2 passes through the points (–2,3) and (–6, *k*).
The value of *k* is

A 11

B -5

C 1

D -13

2. Which of the following gives equal roots for the equation $x^2 + kx + 8 = 0$

A $k = 8\sqrt{2}$

B $k = 32$

C $k = 6$

D $k = 4\sqrt{2}$

3. The **rate of change** of the function $f(x) = \dfrac{3}{2x^2}$ when $x = 1$ is

A -12

B 3

C $\frac{3}{2}$

D -3

4. Given that $\cos A = \frac{\sqrt{2}}{5}$, the exact value of $\sin A$ is

A $\dfrac{\sqrt{23}}{5}$

B $-\frac{17}{25}$

C $\dfrac{\sqrt{23}}{25}$

D cannot be found

5. $\int_0^{2a} 4x\ dx$ is

A 4

B $16a^2$

C $8a^2$

D $4a^2$

6. The gradient of the tangent to the circle $x^2 + y^2 + 6x - 4y - 4 = 0$ at the point $(1,3)$ on the circumference is

A $\frac{1}{4}$

B -4

C $-\frac{1}{4}$

D $\frac{2}{5}$

7. If $f(x) = 3x$ and $g(x) = x^2 - 6x$ then $g(f(1))$ is equal to

A -15

B -9

C 0

D 3

8. **Three** of the following are equal in value. Which is the exception?

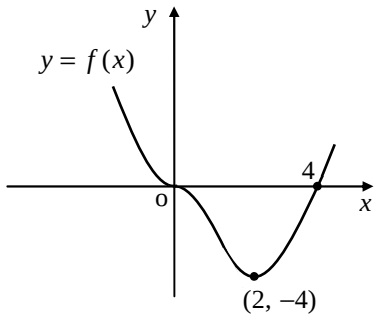
A $\cos\frac{\pi}{3}$

B $\sin\frac{5\pi}{6}$

C $\cos\frac{4\pi}{3}$

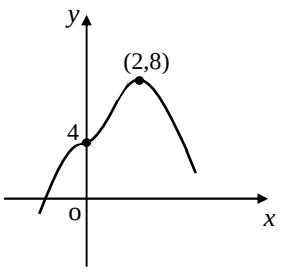
D $-\sin\frac{11\pi}{6}$

9. Part of the graph of the function $y = f(x)$ is shown opposite.

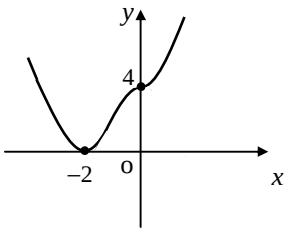


Which of the following graphs represents the related function $y = f(-x) + 4$?

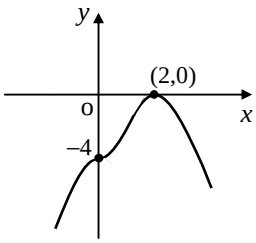
A



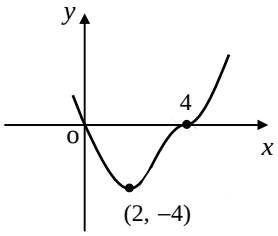
B



C



D



10. PQRS is a parallelogram. P, Q and R have coordinates (–4, –3), (–1,5) and (7,7) respectively.

The coordinates of S are

A (–11, –1)

B (4,0)

C (4, –1)

D (15,9)

11. A recurrence relationship is defined as $U_{n+1} = 0 \cdot 6U_n + b$.

If the limit of the recurrence relationship is 20, the value of *b* is

A 12

B 50

C $33\frac{1}{3}$

D 8

12. The minimum turning value of the function $f(x) = x^2 - 4x + 8$ is

A 4

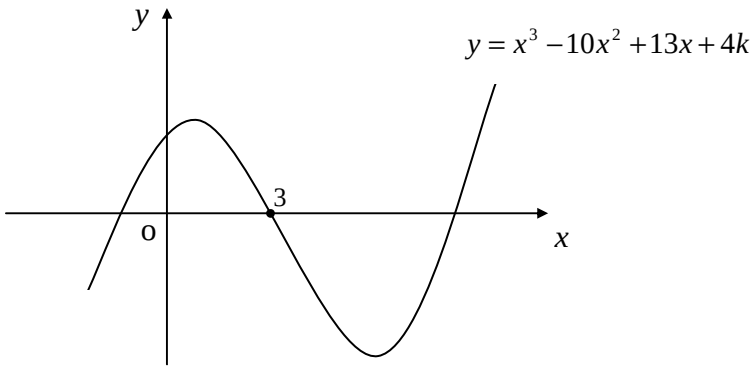
B 2

C 8

D -4

13. The diagram below shows part of the curve with equation $y = x^3 - 10x^2 + 13x + 4k$.

The curve crosses the *x*-axis at (3,0).



The value of *k* is

A 12

B 6

C 0

D -6

14. If $a^{\frac{1}{2}} = \sqrt{8}\, b^{\frac{3}{2}}$ then *b* equals

A $\dfrac{a^{\frac{1}{3}}}{\sqrt{8}}$

B $\dfrac{a^{\frac{1}{3}}}{2}$

C $\dfrac{a^{\frac{3}{4}}}{\sqrt{8}}$

D $\dfrac{a^{\frac{1}{3}}}{8}$

15. The equation of the circle, centre (–5,6) and having the *x*-axis as a tangent is

A $(x+5)^2+(y-6)^2=25$

B $(x-5)^2+(y+6)^2=25$

C $(x+5)^2+(y-6)^2=36$

D $(x-5)^2+(y+6)^2=36$

16. If $\cos\theta = -\frac{1}{\sqrt{2}}$, for $0 \leq \theta \leq 2\pi$, the value of θ is,

A $\frac{\pi}{4}$

B $-\frac{\pi}{4}$

C $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

D $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

17. The range of values of *x* for which the function $f(x)=x^2+8x-12$ is decreasing is

A $x<-4$

B $x>-4$

C $x<4$

D $x>4$

18. A function is defined as $f(x)=3x^3-27x$.

It has a stationary point at

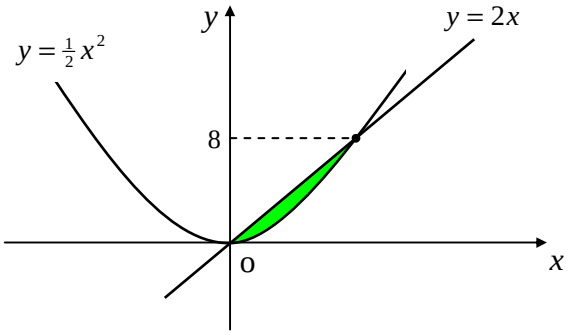
A $x=\sqrt{3}$ and $x=-\sqrt{3}$

B only $x=3$

C only $x=\sqrt{3}$

D $x=3$ and $x=-3$

19. The diagram, **which is not drawn to scale**, shows parts of the graphs of the curve $y = \frac{1}{2}x^2$ and the line $y = 2x$.



The shaded area above can be represented by

A $\int_0^8 \left(2x - \frac{1}{2}x^2\right) dx$

B $\int_0^4 \left(\frac{1}{2}x^2 - 2x\right) dx$

C $\int_0^4 \left(2x - \frac{1}{2}x^2\right) dx$

D $\int_0^8 \left(\frac{1}{2}x^2 + 2x\right) dx$

20. Given that $\sin x = a^2 - 1$ and $\cos x = a - 1$ then $\tan x$ equals

A a

B $a + 1$

C $a^3 - a^2 - a + 1$

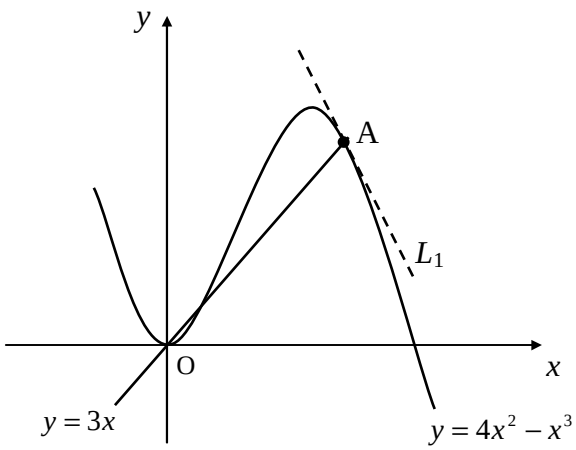
D $\dfrac{1}{a+1}$

[END OF SECTION A]

SECTION B
ALL questions should be attempted

21. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve $y = 4x^2 - x^3$ and the straight line $y = 3x$.

The line intersects the curve at three points.



(a) Find the coordinates of the point A. **4**

(b) The line L_1 is the tangent to the curve at A.

Establish the equation of the line L_1. **4**

22. A function is given by the equation, $x^3 + px - 6 = 0$, where p is a constant.

(a) Given that x = -2 is a root of this function, find the value of p. **3**

(b) Hence, solve the function fully, given that $x > 0$. **4**

23. An equation in x is given as

$$\frac{5x}{x^2 - k^2} = \frac{4}{x + k^2}\ ,\ \text{where } \boldsymbol{k} \textbf{ is a constant and non-zero}.$$

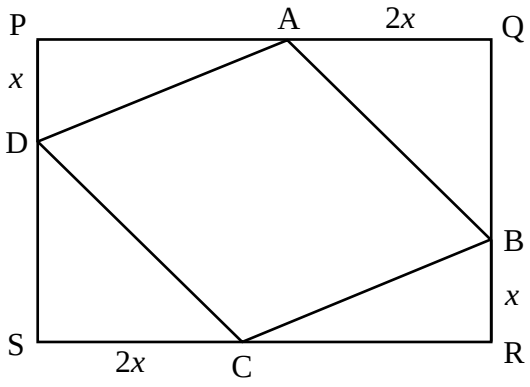
(a) Show that this equation can be written as

$$x^2 + 5k^2x + 4k^2 = 0 \qquad \textbf{2}$$

(b) Hence find the **two** values of k for which the equation $x^2 + 5k^2x + 4k^2 = 0$ has equal roots. **4**

24. PQRS is a rectangle **measuring 6 units by 4 units**.

Points A, B, C and D are points on the sides of the rectangle such that AQ = SC = $2x$ and PD = BR = x as shown.



(a) Show that the area of ABCD is given by the function $A(x) = 4x^2 - 14x + 24$. **4**

(b) Hence find the value of x which minimises the area of ABCD and calculate this minimum area. **5**

[END OF SECTION B]

[END OF QUESTION PAPER]