Dingwall Academy

NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2007/2008
Paper 1
Assessing Units 1 & 2

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2007/2008 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

Sample Question

A line has equation $y = 4x - 1$.

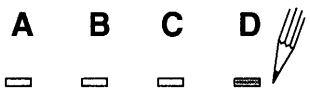
If the point $(k,7)$ lies on this line, the value of k is

A 2
B 27
C 1·5
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).

Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

SECTION A

ALL questions should be attempted

1. A line has as its equation $3y = x + 6$.
Any line parallel to this line will have as its gradient

A -3
B 1
C $-\frac{1}{3}$
D $\frac{1}{3}$

2. If $f(x) = \frac{1}{x^3}$ and $x \neq 0$, then $f'(x)$ is

A $\frac{1}{3x^2}$
B $-\frac{3}{x^4}$
C $-\frac{3}{x^2}$
D $-\frac{1}{2x^2}$

3. The remainder when $2x^3 + x^2 - 1$ is divided by $x - 2$ is

A 9
B 5
C 19
D -13

4. Which of the following is/are true of the circle with equation $x^2 + y^2 - 36 = 0$?

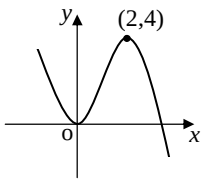
1 It passes through the origin.
2 It has a radius of 6.
3 It has the origin as its centre.

A 1 only
B 2 only
C 2 and 3 only
D some other combination of responses

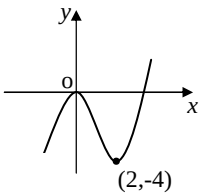
5. If $\sin\theta = \cos\theta$ then $\tan\theta$ equals

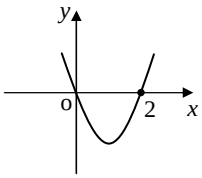
A 0
B -1
C 1
D unknown

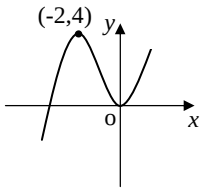
6. Part of the graph of $y = f(x)$ is shown below.

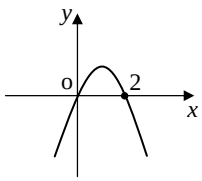


The graph of $y = f'(x)$ could be represented by

A


B


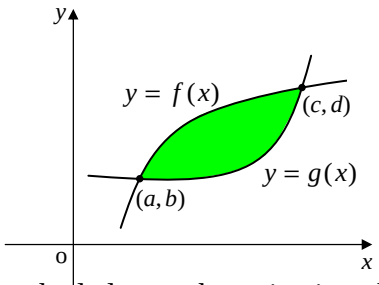
C


D


7. A and B are points (1, p) and (2,4) respectively on the curve with equation $y = x^3 - 4$.
The gradient of the line AB is

A 7
B $\frac{1}{7}$
C 5
D 1

8.



The shaded area above is given by

A $\int_b^d \big(f(x) - g(x)\big)\,dx$
B $\int_a^c \big(f(x) + g(x)\big)\,dx$
C $\int_a^c \big(f(x) - g(x)\big)\,dx$
D $\int_a^d \big(f(x) - g(x)\big)\,dx$

9. Two functions, defined on suitable domains, are given as $f(x) = 3x^2 - 2$ and $g(x) = 1 - x$. The value of $f(g(2))$ is

A -9
B -5
C -1
D 1

10. The value of $\cos\frac{5\pi}{6}$ is

A $-\frac{1}{2}$
B $-\frac{\sqrt{3}}{2}$
C $\frac{\sqrt{3}}{2}$
D $\frac{1}{2}$

11. The tangent to the curve with equation $y = 2x^3 - 1$ at the point where $x = 1$ has gradient

A 1
B 6
C 5
D $-\frac{1}{2}$

12. A circle has as its equation $x^2 + y^2 + 4x - 2y - 4 = 0$. Which of the following correctly states the coordinates of its centre and the value of its radius?

A $(-2,1)$, $r = 1$
B $(2,-1)$, $r = 3$
C $(-2,1)$, $r = 3$
D $(2,-1)$, $r = 1$

13. $\int_1^2 3x^2\ dx$ is equal to

A 6
B 5
C 9
D 7

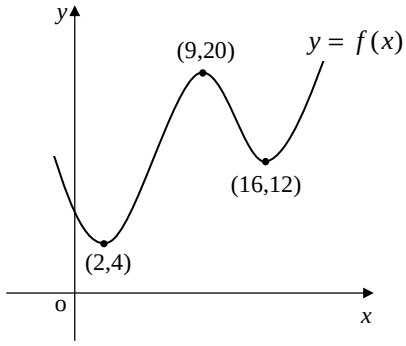
14. A recurrence relation is defined by $U_{n+1} = 0 \cdot 4U_n - 24$.
The limit of this sequence is

A -40
B -24
C $0 \cdot 03$
D 50

15. If x and y are integers the value of $(x+y)^2 - (x-y)^2$ is always

A negative
B positive
C a perfect square
D a multiple of 4

16.



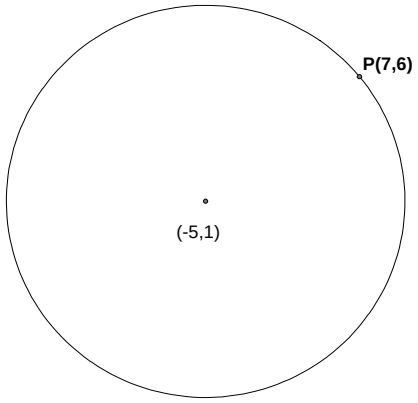
The diagram shows part of the graph of $y = f(x)$.

Which of the following is/are true for the function above?

1 $f'(0) < 0$
2 $f'(6) < 0$
3 $f'(9) = 0$
4 $f'(12) > 0$

A 2 and 3 only
B 3 only
C 1 and 3 only
D 1, 2, 3 and 4

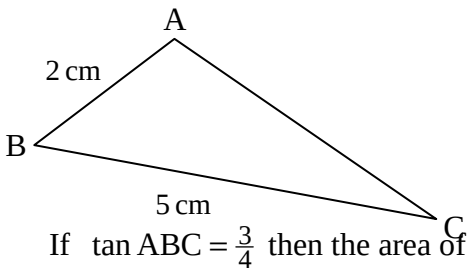
17. The point P(7,6) lies on a circle with centre (-5,1) as shown in the diagram.



What is the length of the diameter?

A $2\sqrt{53}$ units
B $2\sqrt{111}$ units
C $2\sqrt{157}$ units
D 26 units

18.



If $\tan \text{ABC} = \frac{3}{4}$ then the area of triangle ABC in square centimetres is

A 5
B 4
C $\frac{15}{4}$
D 3

19. The quadratic equation $4kx^2 - 8x + k = 0$ has equal roots.
The value of k, where $k > 0$ is

A 4
B 2
C 0
D -2

[END OF SECTION A]

20. $f(x) = ax^2 - 2x - 5$ has a stationary value when $x = 3$.
The value of a is

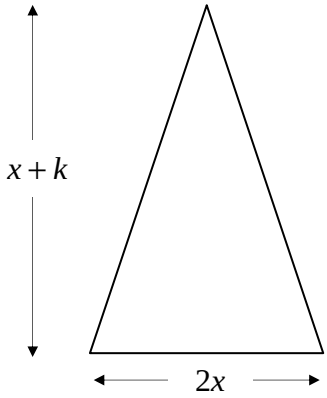
A $\frac{1}{3}$
B $-\frac{1}{3}$
C $\frac{7}{6}$
D $\frac{11}{9}$

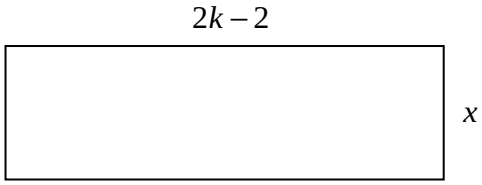
SECTION B

ALL questions should be attempted

21. Consider the isosceles triangle and the rectangle below.

The triangle has a base measuring $2x$ and a vertical height of $x + k$.
The rectangle has dimensions $2k - 2$ by x as shown.
All dimensions are in centimetres.





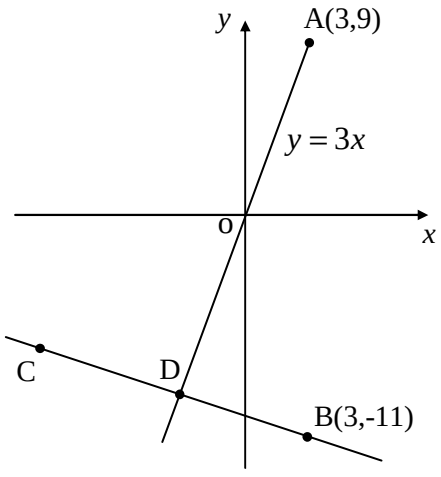
(a) Given that the **area of the rectangle** is 4cm^2 **more than** the area of the triangle, **show clearly** that the following equation can be formed.

$$x^2 + (2-k)x + 4 = 0$$ **3**

(b) Hence find k, given that the equation $x^2 + (2-k)x + 4 = 0$ has equal roots and $k > 0$. **3**

(c) Find x when k takes this value and calculate the area of each shape. **3**

22. In the diagram A has coordinates (3,9) and the point B has coordinates (3,-11) as shown.
A lies on the line with equation $y = 3x$.



(a) If line BC is perpendicular to the line AD, establish the equation of BC. **2**

(b) Hence find the coordinates of D. **3**

(c) If D is the mid-point of BC, write down the coordinates of C. **1**

(d) Find the equation of the circle passing through the points A, D and C. **4**

23. A function is defined on a suitable domain as $f(x) = \frac{1}{3}x^3 - 4x^2 + x$.

(a) Show that its derivative can be expressed in the form

$f'(x) = (x + p)^2 + q$, and state the values of p and q. **4**

(b) Hence state the minimum rate of change of this function and the corresponding value of x. **2**

24. Find the solution(s) of the equation $2\cos^2 a = \cos a + 1$ for $0 \leq a \leq \pi$. **5**

[END OF SECTION B]

[END OF QUESTION PAPER]