Polynomials Past Papers Unit 2 Outcome 1

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Given $p(x) = x^2 + x - 6$, which of the following are true?

   I. $(x + 3)$ is a factor of $p(x)$.

   II. $x = 2$ is a root of $p(x) = 0$.

   A. Neither I nor II is true

   B. Only I is true

   C. Only II is true

   D. Both I and II are true

2. When $2ax^3 + (a + 1)x - 6$ is divided by $x + 2$, the remainder is 2.

   What is the value of a?

   A. $\frac{5}{3}$

   B. $-\frac{4}{9}$

   C. $-\frac{5}{9}$

   D. $-\frac{5}{7}$

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions

[SQA]   3. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

   (*b*) On the same diagram sketch:
   (i) the graph of $y = f(x)$;
   (ii) the graph of $y = 10 - f(x)$. **4**

   (*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

[SQA]   4. Find the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing. **4**

[SQA]   5.

   (i) Write down the condition for the equation $ax^2 + bx + c = 0$ to have no real roots. **1**

   (ii) Hence or otherwise show that the equation $x(x + 1) = 3x - 2$ has no real roots. **2**

[SQA]   6. Show that the roots of the equation $(k - 2)x^2 - (3k - 2)x + 2k = 0$ are real. **4**

hsn.uk.net

[SQA] 7. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots?   **3**

[SQA] 8. Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers.   **5**

[SQA] 9. Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots.   **2**

[SQA] 10. For what value of a does the equation $ax^2 + 20x + 40 = 0$ have equal roots?   **2**

[SQA] 11. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k.   **5**

[SQA] 12. The roots of the equation $(x - 1)(x + k) = -4$ are equal. Find the values of k.   **5**

[SQA] 13. (a) $f(x) = 2x + 1$, $\quad g(x) = x^2 + k$, where k is a constant.
　　　　　　(i)　Find $g(f(x))$.   **(2)**
　　　　　　(ii)　Find $f(g(x))$.   **(2)**

　　　　　(b) (i)　Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to
　　　　　　　　$2x^2 + 4x - k = 0$.   **(2)**
　　　　　　(ii)　Determine the nature of the roots of this equation when $k = 6$.   **(2)**
　　　　　　(iii)　Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots.   **(3)**

[SQA] 14. Factorise fully $2x^3 + 5x^2 - 4x - 3$.   **4**

[SQA] 15. Find p if $(x + 3)$ is a factor of $x^3 - x^2 + px + 15$.   **3**

[SQA] 16. (a) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$.   **1**
　　　　　(b) Hence find the other roots.   **3**

[SQA] 17. Find k if $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$. **3**

[SQA] 18. When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x - 2)$, the remainder is 114.

One factor of $f(x)$ is $(x + 1)$.

Find the values of p and q. **5**

[SQA] 19. One root of the equation $2x^3 - 3x^2 + px + 30 = 0$ is -3.

Find the value of p and the other roots. **4**

[SQA] 20. O (a) Show that $(x - 3)$ is a factor of $f(x)$ where $f(x) = 2x^3 + 3x^2 - 23x - 12$. **2**

x
y (b) Hence express $f(x)$ in its fully factorised form. **2**

[SQA] 21. Express $x^4 - x$ in its fully factorised form. **4**

O
[SQA] 22. x (a) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$. **2**
y (b) Show algebraically that there are no other real roots. **3**

[SQA] 23. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **4**

[SQA] 24. (a) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

[SQA] 25.

(a) Write the equation $\cos 2\theta + 8 \cos \theta + 9 = 0$ in terms of $\cos \theta$ and show that, for $\cos \theta$, it has equal roots. **3**

(b) Show that there are no real roots for θ. **1**

[SQA] 26. Calculate the least positive integer value of k so that the
graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis.

4

$y = kx^2 - 8x + k$

(0,k)

frag replacements

O
x
y
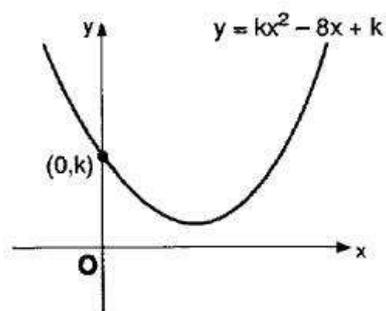
[SQA] 27. (a) The point A(2, 2) lies on the parabola
$y = x^2 + px + q$.
Find a relationship between p and q.

(1)

A (2, 2)

O
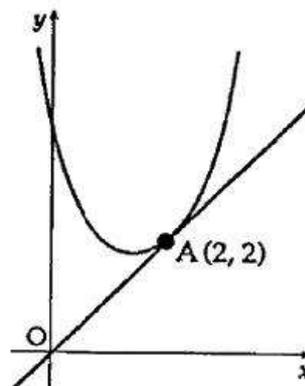
(b) The tangent to the parabola at A is the line $y = x$. Find the value of p.
Hence find the equation of the parabola.

(6)

frag replacements (c) Using your answers for p and q, find the value of the discriminant of
O $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this
x value ?
y

(2)

[SQA] 28. The diagram shows part of the graph of the
curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.

$y = f(x)$

(a) Find the x-coordinate of the maximum
turning point.

5

(b) Factorise $2x^3 - 7x^2 + 4x + 4$.

3

(c) State the coordinates of the point A and
hence find the values of x for which
$2x^3 - 7x^2 + 4x + 4 < 0$.

2

A

O   (2,0) x

replacements

O
x
y **hsn**.uk.net
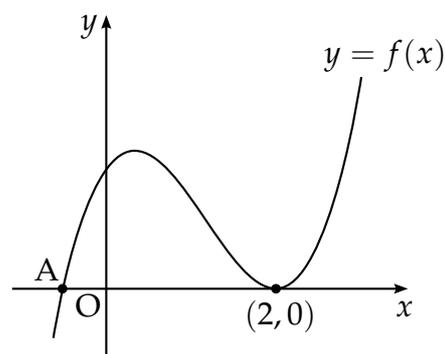
[SQA]   29.

    (a) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.

        The function g is defined by $g(x) = x - 1$.

        Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$.   **4**

    (b) Factorise fully $f(g(x))$.   **3**

    (c) The function k is such that $k(x) = \dfrac{1}{f(g(x))}$.

   For what values of x is the function k not defined?   **3**

[SQA]   30.   The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

    (a) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis.   **(3)**

    (b) Find algebraically the coordinates of the other points at which the curve crosses the x-axis.   **(4)**

[SQA]   31.   The diagram shows part of the graph of the curve with equation
$f(x) = x^3 + x^2 - 16x - 16$.
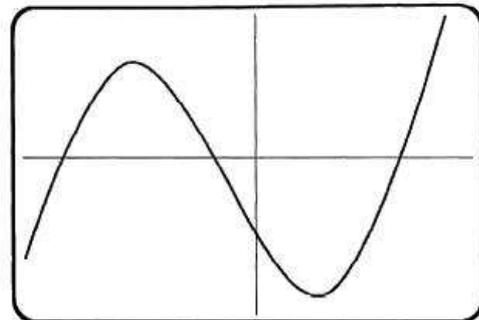
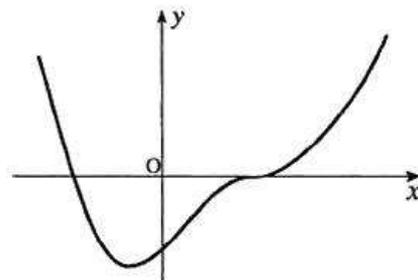   (a) Factorise $f(x)$.   **(3)**

    (b) Write down the co-ordinates of the four points where the curve crosses the x and y axes.   **(2)**
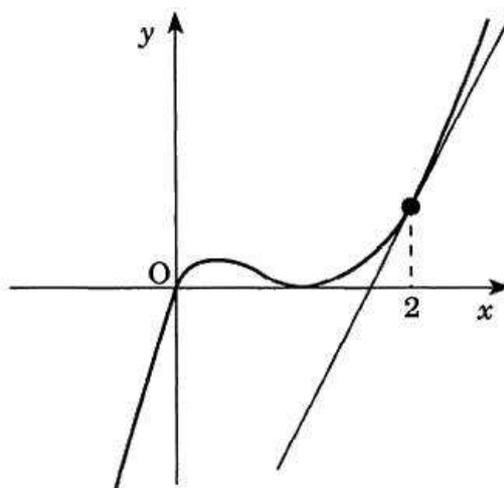
    (c) Find the turning points and justify their nature.   **(6)**

[SQA]   32.   The function f, whose incomplete graph is shown in the diagram, is defined by
$f(x) = x^4 - 2x^3 + 2x - 1$.
Find the coordinates of the stationary points and justify their nature.   **(8)**

hsn.uk.net

[SQA] 33. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.


(a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(b) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

hsn.uk.net

[SQA] 34. The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.
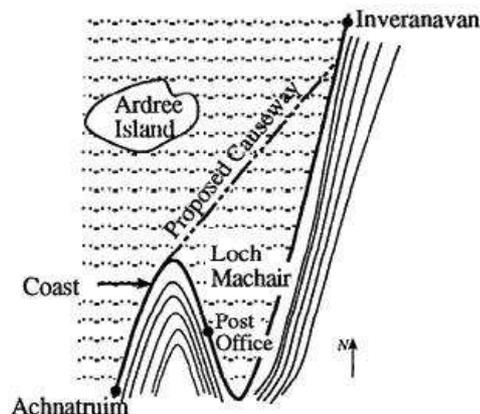

With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.
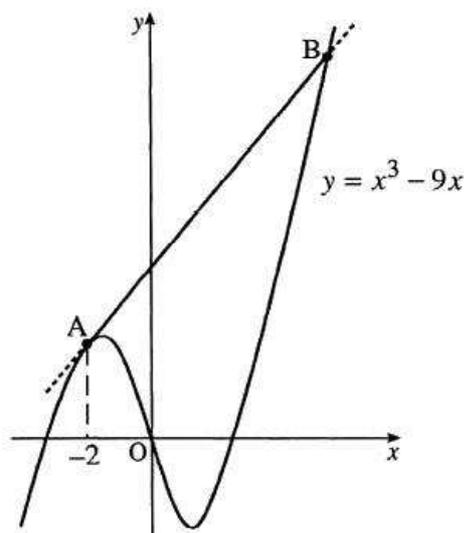
(a) (i) Write down the coordinates of A.   (5)
     (ii) Find the equation of the line AB.
(b) Determine the coordinates of the point B which represents the northern end   (7)
     of the causeway.

[SQA] 35. The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.
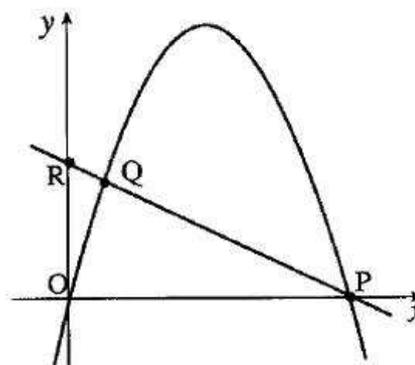
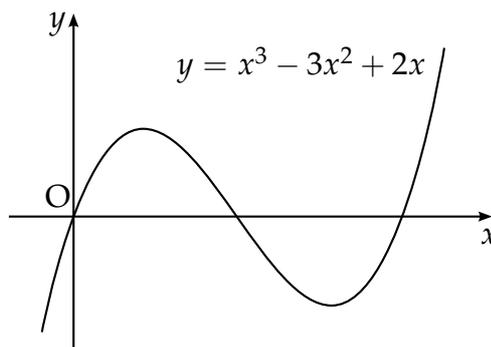(a) Find the coordinates of the point P.   2
(b) R is the point (0, 2). Find the equation of PR.   2
(c) The line and the parabola also intersect at Q. Find the   4
     coordinates of Q.

[SQA]   36. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.

(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. PSfrag replacements

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again.

y

$y = x^3 - 3x^2 + 2x$

O

x

5

5

[END OF WRITTEN QUESTIONS]