Differentiation Past Papers Unit 1 Outcome 3

1. Differentiate $2\sqrt[3]{x}$ with respect to x.

   A. $6\sqrt{x}$

   B. $\frac{3}{2}\sqrt[3]{x}^4$

   C. $-\dfrac{4}{3\sqrt[3]{x}^2}$

   D. $\dfrac{2}{3\sqrt[3]{x}^2}$ **2**

[SQA] 2. Given $f(x) = 3x^2(2x-1)$, find $f'(-1)$. **3**

[SQA] 3. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of $45°$ with the positive direction of the x-axis. **4**

[SQA] 4. If $y = x^2 - x$, show that $\dfrac{dy}{dx} = 1 + \dfrac{2y}{x}$. **3**

[SQA] 5. Find $f'(4)$ where $f(x) = \dfrac{x-1}{\sqrt{x}}$. **5**

[SQA] 6. Find $\dfrac{dy}{dx}$ where $y = \dfrac{4}{x^2} + x\sqrt{x}$. **4**

[SQA] 7. If $f(x) = kx^3 + 5x - 1$ and $f'(1) = 14$, find the value of k. **3**

[SQA] 8. Find the x-coordinate of each of the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ at which the tangent is parallel to the x-axis. **4**

[SQA] 9. Calculate, to the nearest degree, the angle between the x-axis and the tangent to the curve with equation $y = x^3 - 4x - 5$ at the point where $x = 2$. **4**

[SQA] 10. The point $P(-1,7)$ lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P. **4**

[SQA] 11. Find the equation of the tangent to the curve $y = 4x^3 - 2$ at the point where $x = -1$. **4**

[SQA] 12. Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$. **4**

[SQA] 13. Find the equation of the tangent to the curve $y = 3x^3 + 2$ at the point where $x = 1$. **4**

[SQA] 14. The point $P(-2, b)$ lies on the graph of the function $f(x) = 3x^3 - x^2 - 7x + 4$.

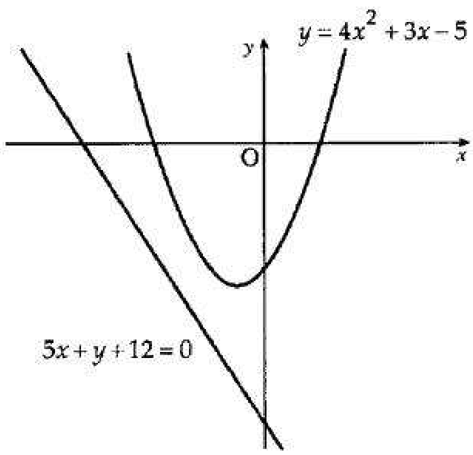
(*a*) Find the value of b. **1**

(*b*) Prove that this function is increasing at P. **3**

[SQA] 15. For what values of x is the function $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$ increasing? **5**

[SQA] 16. Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1 + \frac{dy}{dx}\right) = 2y$. **3**

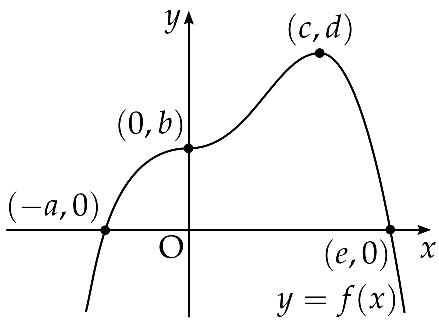
[SQA] 17. The diagram below shows a parabola with equation $y = 4x^2 + 3x - 5$ and a straight line with equation $5x + y + 12 = 0$.
A tangent to the parabola is drawn parallel to the given straight line.
Find the x-coordinate of the point of contact of this tangent. **5**



[SQA] 18. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

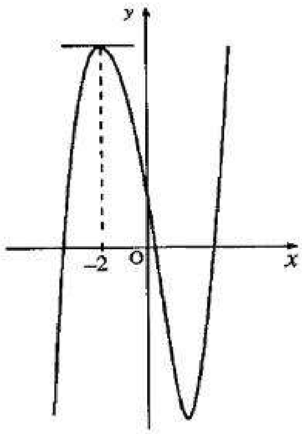
There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

Sketch the graph of the derived function f'. **3**



[SQA] 19. The diagram shows a sketch of the curve $y = x^3 + kx^2 - 8x + 3$. The tangent to the curve at $x = -2$ is parallel to the x-axis.

Find the value of k. **4**



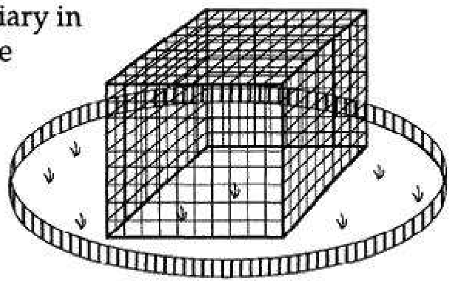
[SQA] 20. A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbb{R}$.

(*a*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **3**

(*b*) Find the stationary points of this curve $y = f(x)$ and determine their nature. **7**

(*c*) Sketch the curve $y = f(x)$. **2**

[SQA] 21. The owners of a zoo intend to build a new aviary in the shape of a cuboid with a square floor. The volume of the aviary will be 500 m^3.



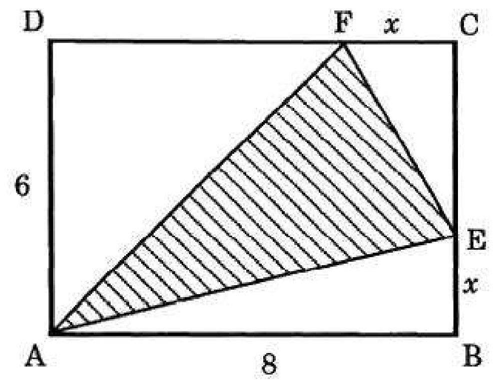
(*a*) If x metres is the length of one edge of the floor, show that the area A square metres of netting required is given by

$$A = x^2 + \frac{2000}{x}.$$

(4)

(*b*) Find the dimensions of the aviary to ensure that the cost of netting is minimised. **(6)**

[SQA] 22. An yacht club is designing its new flag.
The flag is to consist of a red triangle on a yellow rectangular background.
In the yellow rectangle ABCD, AB measures 8 units and AD is 6 units. E and F lie on BC and CD, x units from B and C as shown in the diagram.

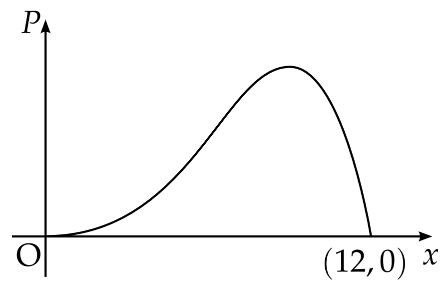


(*a*) Show that the area, H square units, of the red triangle AEF is given by $H(x) = 24 - 4x + \frac{1}{2}x^2$. **(4)**

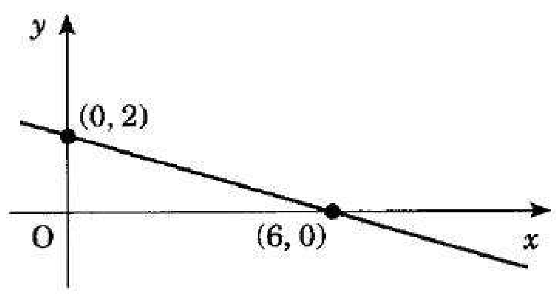
(*b*) Hence find the greatest and least possible values of the area of triangle AEF. **(8)**

[SQA] 23. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model , illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.

Find the value of x which gives the maximum profit. **5**



[SQA] 24. The straight line shown in the diagram has equation $y = f(x)$.
Determine $f'(x)$. **2**



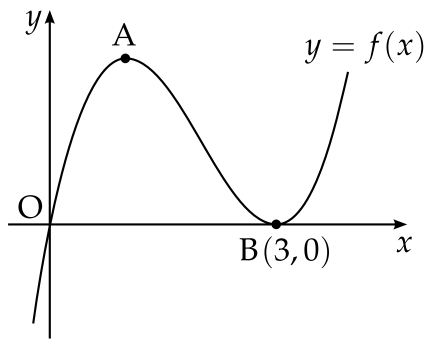
[SQA] 25. A ball is thrown vertically upwards. The height h metres of the ball t seconds after it is thrown, is given by the formula $h = 20t - 5t^2$.

(*a*) Find the speed of the ball when it is thrown (i.e. the rate of change of height with respect to time of the ball when it is thrown). **3**

(*b*) Find the speed of the ball after 2 seconds.
Explain your answer in terms of the movement of the ball. **2**

[SQA] 26. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at $B(3, 0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

[SQA] 27. A curve has equation $y = x^4 - 4x^3 + 3$.

(*a*) Find algebraically the coordinates of the stationary points. **6**

(*b*) Determine the nature of the stationary points. **2**

[SQA] 28. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.

Prove that this curve has no stationary points. **5**

[SQA] 29. Find the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing. **4**

[SQA] 30. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

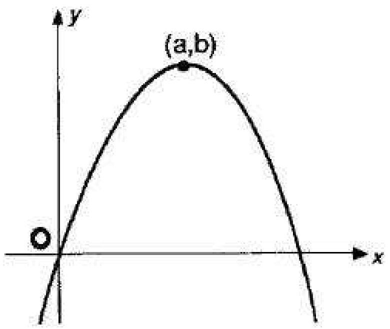
[SQA] 31. A ball is thrown vertically upwards.

After t seconds its height is h metres, where $h = 1{\cdot}2 + 19{\cdot}6t - 4{\cdot}9t^2$.

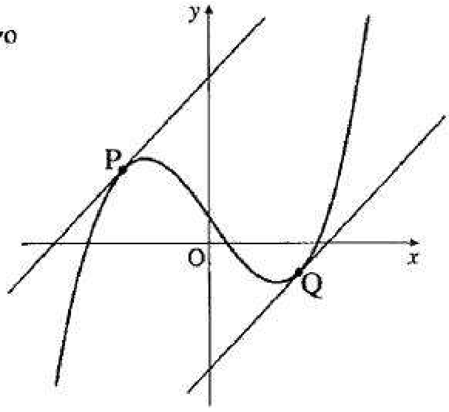
(*a*) Find the speed of the ball after 1 second. **3**

(*b*) For how many seconds is the ball travelling upwards? **2**

[SQA] 32. The line with equation $y = x$ is a tangent at the origin to the parabola with equation $y = f(x)$. The parabola has a maximum turning point at (a, b).
Sketch the graph of $y = f'(x)$. **4**



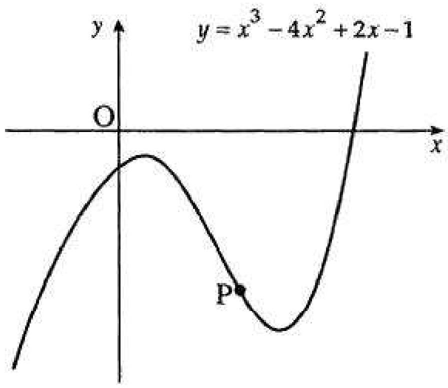
[SQA] 33. The diagram shows a sketch of the graph of $y = x^3 - 9x + 4$ and two parallel tangents drawn at P and Q.



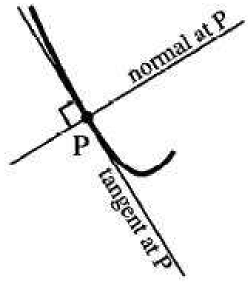
(a) Find the equations of the tangents to the curve $y = x^3 - 9x + 4$ which have gradient 3. **6**

(b) Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$. **6**

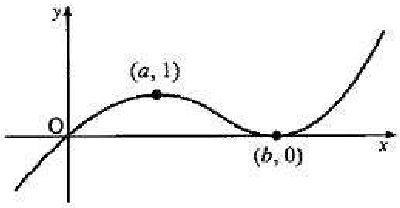
[SQA] 34. (a) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$.
Find the equation of the tangent to the curve at the point P where $x = 2$. **(5)**



(b) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P.
Find the angle which the normal at P makes with the positive direction of the x-axis. **(2)**



[SQA] 35. A sketch of the graph of the cubic function f is shown. It passes through the origin, has a maximum turning point at $(a, 1)$ and a minimum turning point at $(b, 0)$.



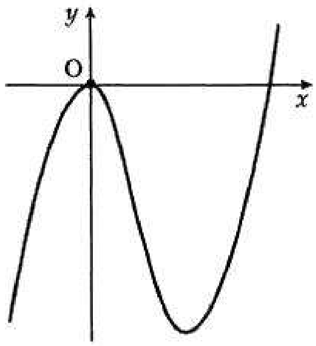
(a) Make a copy of this diagram and on it sketch the graph of $y = 2 - f(x)$, indicating the coordinates of the turning points. 3

(b) On a separate diagram sketch the graph of $y = f'(x)$. 2

(c) The tangent to $y = f(x)$ at the origin has equation $y = \frac{1}{2}x$. Use this information to write down the coordinates of a point on the graph of $y = f'(x)$. 1

[SQA] 36. (a) The diagram shows a part of the curve with equation $y = 2x^2(x-3)$. Find the coordinates of the stationary points on the graph and determine their nature. **(5)**

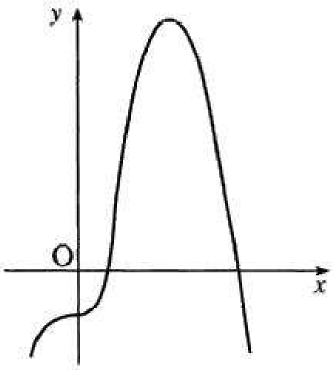
(b) State the range of values of k for which $y = k$ intersects the graph in three distinct points. **(2)**



[SQA] 37. A curve has equation $y = -x^4 + 4x^3 - 2$. An incomplete sketch of the graph is shown in the diagram.

(a) Find the coordinates of the stationary points. **(6)**

(b) Determine the nature of the stationary points. **(2)**



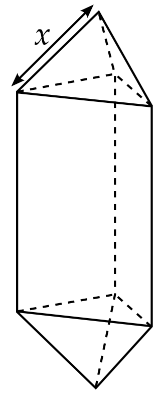
[SQA] 38. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

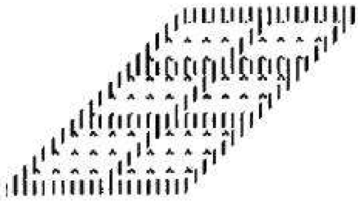
$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

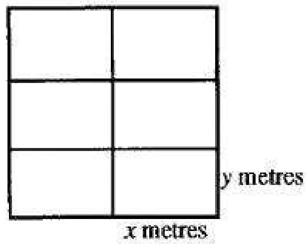
where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**



[SQA] 39. A zookeeper wants to fence off six individual animal pens.





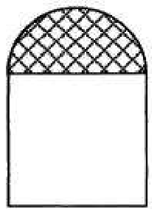
Each pen is a rectangle measuring x metres by y metres, as shown in the diagram.

(a) (i) Express the total length of fencing in terms of x and y.

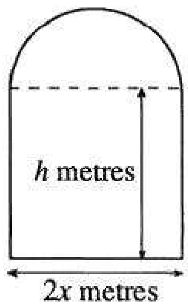
(ii) Given that the total length of fencing is 360m, show that the total area, A m^2, of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$. **4**

(b) Find the values of x and y which give the maximum area and write down this maximum area. **6**

[SQA] 40. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.
The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures $2x$ metres by h metres.

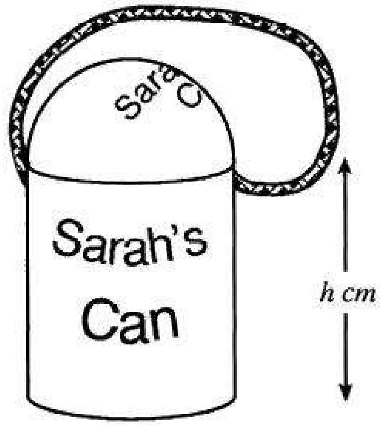


(a) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(b) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

[SQA] 41. A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.

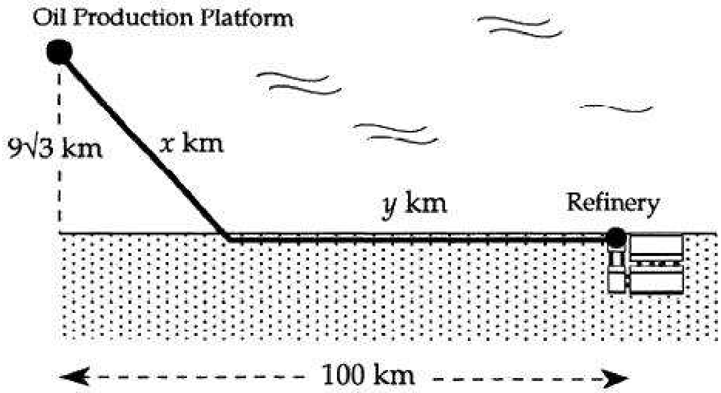


(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \dfrac{800}{r}$. **(3)**

Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

[SQA] 42. An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.



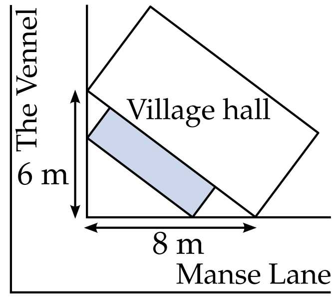
The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(*a*) Show that the total cost of this pipeline is £$C(x)$ million where

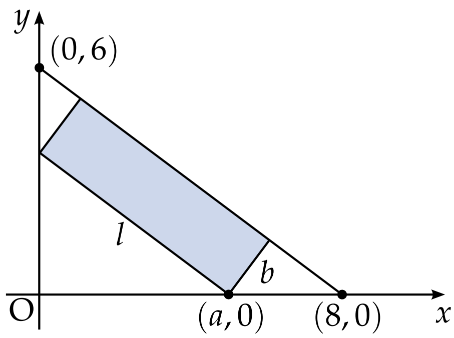
$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$ **(3)**

(*b*) Show that $x = 18$ gives a minimum cost for this pipeline.
Find this minimum cost and the corresponding total length of the pipeline. **(7)**

[SQA] 43. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(a) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8 - a)$. **3**

(b) Find the value of a which produces the largest area of the extension. **4**

[SQA] 44. Diagram 1 is an artist's impression of a new warehouse based on the architect's plans. The warehouse is in the shape of a cuboid and is supported by three identical parabolic girders spaced 30 metres apart.
With coordinate axes as shown in Diagram 2, the shape of each girder can be described by the equation $y = 9 - \frac{1}{4}x^2$.

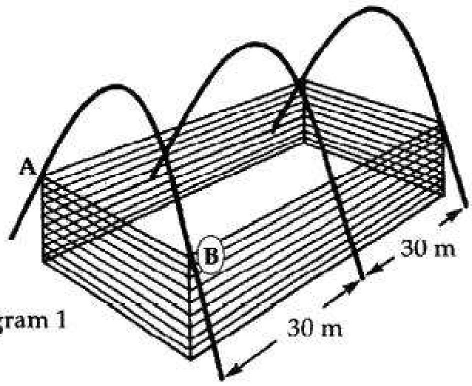


Diagram 1

(a) Given that AB is $2x$ metres long, show that the shaded area in Diagram 2 is $\left(18x - \frac{1}{2}x^2\right)$ square metres. **(2)**

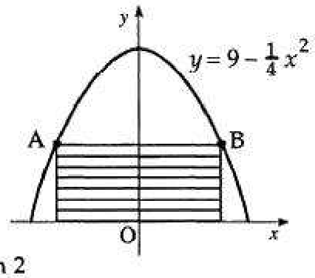
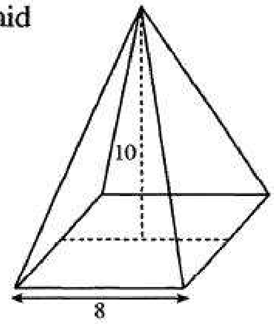


Diagram 2

(b) The architect wished to fit into the girders the cuboidal warehouse which had the maximum volume. Find the value of this maximum volume. **(6)**

[SQA] 45. A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm. and a vertical height of 10cm.

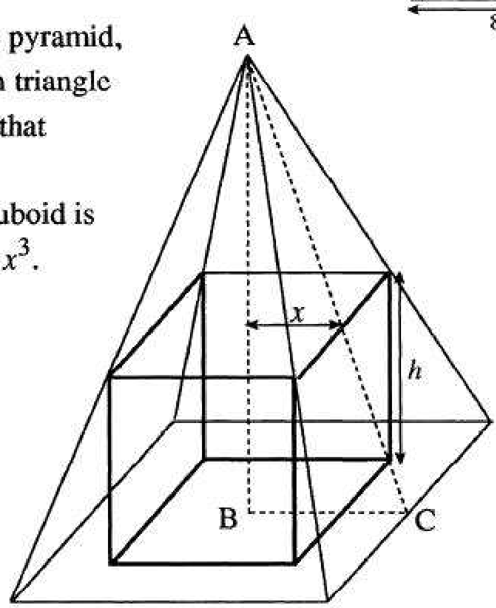


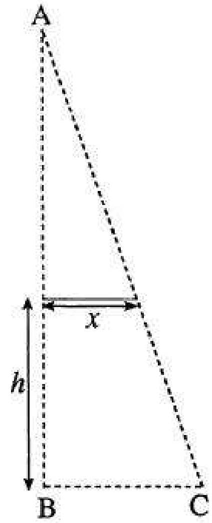
(*a*) The cuboid has a square base of side $2x$ cm and a height of h cm.

If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that

(i) $h = 10 - \frac{5}{2}x$. **(3)**

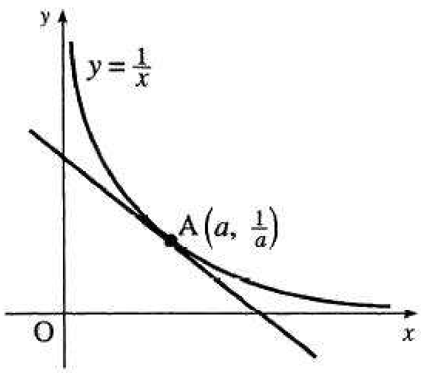
(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$. **(1)**





(*b*) Hence find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **(6)**

[SQA] 46. (*a*) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram. The tangent at A $\left(a, \frac{1}{a}\right)$ has been drawn.
Find the gradient of this tangent. **(4)**



(*b*) Hence show that the equation of this tangent is $x + a^2y = 2a$. **(2)**

(*c*) This tangent cuts the y-axis at B and the x-axis at C.

(i) Calculate the area of triangle OBC **(3)**

(ii) Comment on your answer to c(i). **(1)**

[END OF QUESTIONS]